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Vehicle Suspension System Technology and Design

Avesta Goodarzi
Amir Khajepour

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Vehicle Suspension System Technology and Design

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SYNTHESIS LECTURES IN MECHANICAL ENGINEERING, #02

Vehicle Suspension System Technology and Design

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SYNTHESIS LECTURES IN MECHANICAL ENGINEERING #02

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ABSTRACT

The purpose of this book is to cover essential aspects of vehicle suspension systems and provide an easy approach for their analysis and design. It is intended specifically for undergraduate students and anyone with an interest in design and analysis of suspension systems. In order to simplify the understanding of more difficult concepts, the book uses a step-by-step approach along with pictures, graphs and examples.

The book begins with the introduction of the role of suspensions in cars and a description of their main components. The types of suspensions are discussed and their differences reviewed. The mechanisms or geometries of different suspension systems are introduced and the tools for their analysis are discussed. In addition, vehicle vibration is reviewed in detail and models are developed to study vehicle ride comfort.

KEYWORDS

suspension systems, suspension design, suspension analysis, active and adaptive suspensions, ride comfort

Contents

Preface

This book encompasses all essential aspects of suspension systems and provides an easy approach to their understanding and designing. The book is intended specifically for undergraduate students and is accessible to anyone with an interest in learning about the foundations and design of suspension systems. This book uses a step-by-step approach using pictures, graphs, tables, and examples so that the reader may easily grasp difficult concepts.

After a short introduction, suspension systems and their components are discussed and reviewed. The following sections define and examine suspension mechanisms and their geometrical features. Vehicle ride models are derived to study and discuss vehicle ride comfort. The book ends with analysis on suspension design factors and component sizing.

Acknowledgments

This book would not have been possible without the help of many people. We are particularly grateful to M. Naghibian, M. Ghare, C. Jarvis, S.M. Fard, and A. Pazooki for their assistance in preparing the content of the book, and Ananya Chattoraj for editing and proofreading the book. We are also thankful to Morgan & Claypool Publishers for providing the opportunity for this book and their consistent encouragement and support throughout this project.

Introduction

When people consider purchasing a new vehicle, they normally think of horsepower, torque, 0–100 km/h acceleration, and fuel economy. However, they are likely unaware about an important factor; the engine's power or vehicle speed are utterly useless if the driver cannot control the vehicle. Certainly, many people may recognize the importance of the suspension for ride comfort, but they are less aware about the complete range of vehicle suspension duties. In fact, in addition to ride comfort, the suspension system plays an important role in vehicle performance, stability, and safety. Accordingly, automotive engineers turn their attention to the suspension system, an area usually ignored by customers considering a purchase.

Historic horse-drawn carts had an early form of a suspension system, where the platform swung on iron chains attached to a wheeled frame on the carriage (Figure I.1). This system was the basis for all suspension systems until the end of the 19th century. Obadiah Elliot is known as the first person that used a spring in the suspension system of a vehicle, and Mors of Paris first fitted a suspension system with shock absorbers in 1901. Today, there are many kinds of suspension systems with complex structures and elements, some of which will be discussed here.

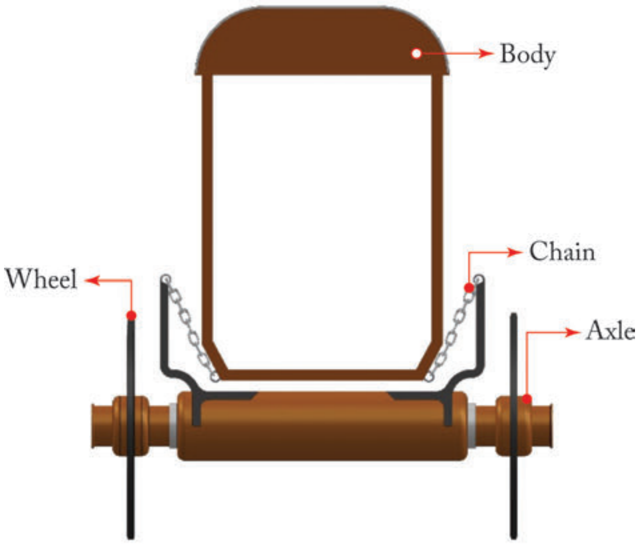


Figure I-1: Early form of suspension systems for horse-drawn carts.

This book begins with the definition of the suspension system and its function. It then goes on to describe the main components and desired features of suspension systems. This is followed by a classification of the different types of suspensions along with their advantages and disadvantages. Major suspension elements like springs, shock absorbers, and the anti-roll-bar are introduced. There will also be a section that presents advanced, active, and semi active suspension systems. Finally, the design and analysis of a suspension mechanism along with its major parts are explained.

CHAPTER 1

Suspensions, Functions, and Main Components

1.1 SUSPENSION SYSTEMS

To many focusing on ride comfort, it may seem like the suspension system is merely a set of springs and shock absorbers which connect the wheels to the vehicle body. However, this is a very simplistic viewpoint of the suspension system. A vehicle suspension system provides a smooth ride over rough roads while ensuring that the wheels remain in contact with the ground and vehicle roll is minimized. The suspension system contains three major parts: a structure that supports the vehicle's weight and determines suspension geometry, a spring that converts kinematic energy to potential energy or vice versa, and a shock absorber that is a mechanical device designed to dissipate kinetic energy.

An automotive suspension connects a vehicle's wheels to its body while supporting the vehicle's weight. It allows for the relative motion between wheel and vehicle body; theoretically, a suspension system should reduce a wheel's degree of freedom (DOF) from 6 to 2 on the rear axle and to 3 on the front axle even though the suspension system must support propulsion, steering, brakes, and their associated forces. The relative motions of the wheels are its vertical movement, rotational movement about the lateral axes, and rotational movement about the vertical axes due to steer angle.

1.2 FUNCTIONS OF SUSPENSION SYSTEMS

As previously mentioned, it is mostly assumed that the only function of a suspension system is the absorption of road roughness; however, the suspension of a vehicle needs to satisfy a number of requirements with partially conflicting aims as a result of different operating conditions. The suspension connects the vehicle's body to the ground, so all forces and moments between the two go through the suspension system. Thus, the suspension system directly influences a vehicle's dynamic behavior. Automotive engineers usually study the functions of a suspension system through three important principle.

- **Ride Comfort:** Ride comfort is defined based on how a passenger feels within a moving vehicle. The most common duty of the suspension system is road isolation—isolating a vehicle body from road disturbances. Generally, ride quality can be quantified by the passenger compartment's level of vibration. There are a lot of inner and outer vibration sources in a vehicle. Inner vibration sources include the vehicle's engine and transmission, whereas road surface irregularities and aerodynamic forces are the outer vibration sources. The spectrum of vibration may be divided up according to ranges in frequency and classified as comfortable (0–25 Hz) or noisy and harsh (25–20,000 Hz).

- **Road Holding:** The forces on the contact point between a wheel and the road act on the vehicle body through the suspension system. The amount and direction of the forces determine the vehicle's behavior and performances, therefore one of the important tasks of the suspension system is road holding. The lateral and longitudinal forces generated by a tire depend directly on the normal tire force, which supports cornering, traction, and braking abilities. These terms are improved if the variation in the normal tire load is minimized. The other function of the suspension is supporting the vehicle's static weight. This task is performed well if the rattle space requirements in the vehicle are kept minimal.

- **Handling:** A good suspension system should ensure that the vehicle will be stable in every maneuver. However, perfect handling is more than stability. The vehicle should respond to the driver's inputs proportionally while smoothly following his/her steering/braking/accelerating commands. The vehicle behavior must be predictable, and behavioral information should accordingly be communicated to the driver. Suspension systems can affect vehicle handling in many ways: they can minimize the vehicle's roll and pitch motion, control the wheels' angles, and decrease the lateral load transfer during cornering.

1.3 MAIN COMPONENTS OF SUSPENSION SYSTEMS

A vehicle suspension system is made of four main components—mechanism, spring, shock absorber, and bushings—as shown in Figure 1.1.

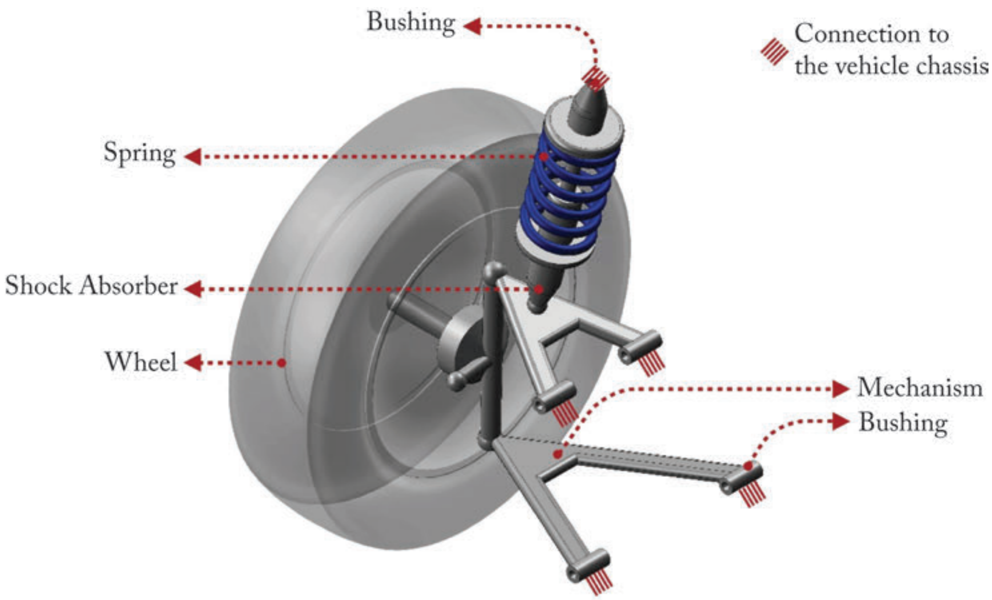


Figure 1.1: The main components of a suspension system.

- **Mechanism:** The suspension mechanism might contain one or several arms that connect a wheel to the vehicle body. They transfer all forces and moments in different directions between the vehicle body and the ground. The mechanism determines some of the most important characteristics of a suspension system. It determines the suspension geometry and wheel angles and their relative motions. Variation in wheel angles during suspension travel causes a change in tire forces, which affect the vehicle's road holding and handling. The main weight of a suspension system arises from its mechanism. Using heavy materials in its construction decreases the ride quality, whereas light materials, although improve ride quality, are more expensive.

- **Spring:** The spring is usually a winding wire or a number of strips of metal that have elastic properties. It supports the vehicle's weight and makes a suspension tolerable for passengers. To best understand suspension behavior, the most important component requiring study is the spring. However, with its importance, a conflict arises: when using high stiffness springs, the vehicle exhibits good road holding and handling but with a noticeably decreased ride comfort. This creates a condition of limitation when choosing an appropriate spring stiffness. The spring weight and size may also make this accommodation difficult.

- **Shock absorber:** The shock absorber is a mechanical or hydraulic device to dampen impulses. A high damping shock absorber compromises the vehicle's ride quality in order to immediately dampen impulses to improve handling and road holding.

- **Bushings:** The bushings prevent the direct contact of two metal objects in order to isolate noise and minimize vibration. Soft materials such as rubber is used in bushings for isolation. In fact, they are a type of vibration isolator used to connect various moving components to the vehicle body or suspension frame. Many types of bushing exist, and they are classified by the number of DOF between the two connected parts that they support. Revolute joints are the most common type of bushings. They are annular cylindrical and support a rotational relative motion, whereas ball joints allow rotational relative motion in all directions. Bushings are some of the most expensive parts in a suspension system.

1.4 DESIRED FEATURES OF SUSPENSION SYSTEMS

A suspension system should satisfy certain requirements for use in vehicles. The main desired features are as follows.

- **Independency:** It is desirable to have the movement of a wheel on one side of the axle to be independent from the movement of the wheel on the other side of the axle. Figure 1.2 (top) shows a vehicle with its left wheel going over a bump. At higher speeds, the wheel can negotiate the bump without disturbing the other wheel. This is only possible when each wheel has an independent suspension. Independency of wheel movement improves a vehicle's ride comfort, road holding, and handling.

- **Good camber control:** The camber angle is the wheel angle about its longitudinal axis (this will be comprehensively discussed in Section 4.1.2). A negative camber is desired since it results in improved handling, however, the convex shape of roads tends toward a positive camber to reduce tire wear. Due to road bump and body roll, the camber will ultimately change. Using a well-designed suspension geometry, we can control the camber angle (Figures 1.3 and 1.4).

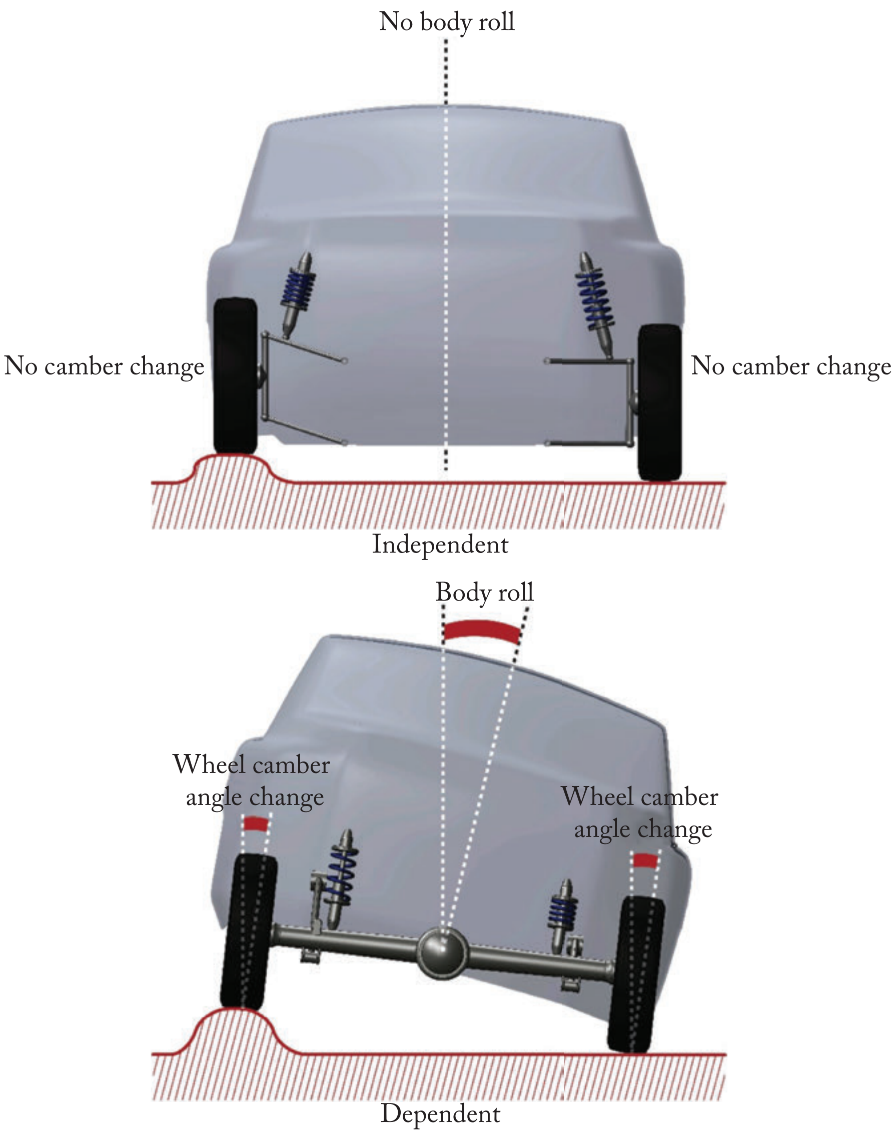


Figure 1.2: Independent vs. dependent suspension systems.

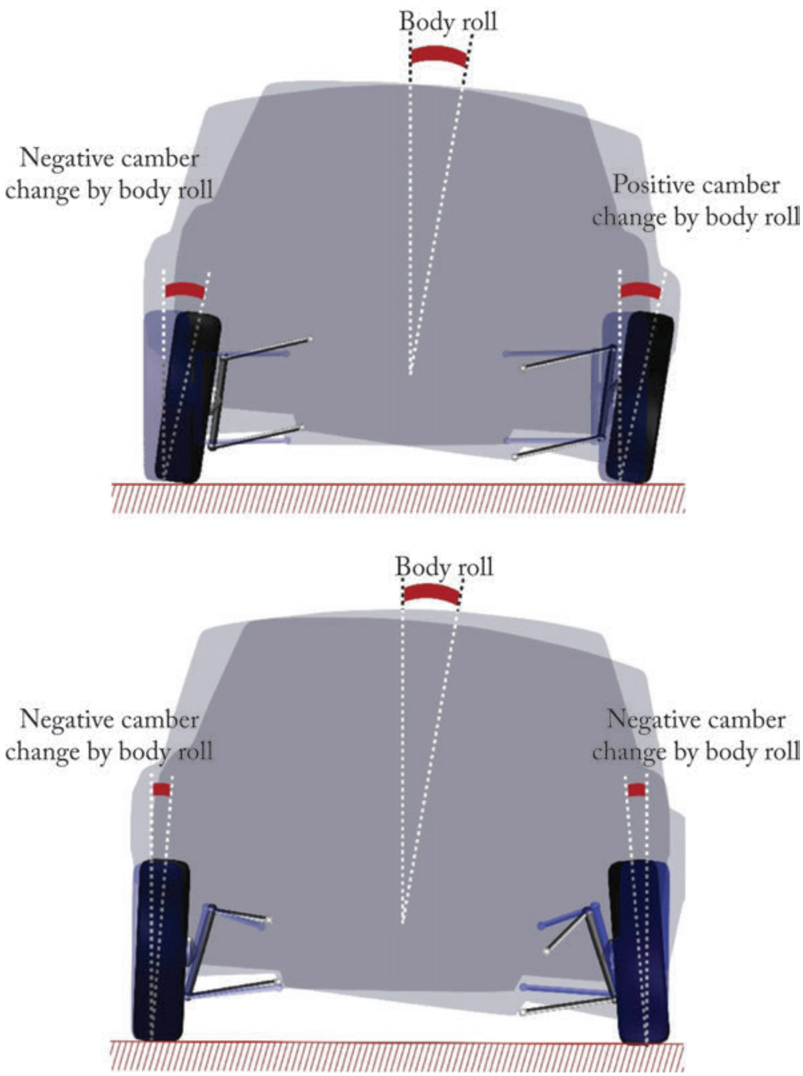


Figure 1.3: Effects of suspension design on camber change due to body roll.

- **Good body roll control:** Each suspension system has a roll center. The hypostatical line that connects the front and rear suspensions' roll centers is called the roll axis. The vehicle's body rolls about this line during cornering maneuvers. It is necessary to analyze the roll axis because of its effect on the body roll and lateral vehicle behavior. The design of the suspension geometry should account for the best location of the roll axis in order to optimize vehicle body roll motion.

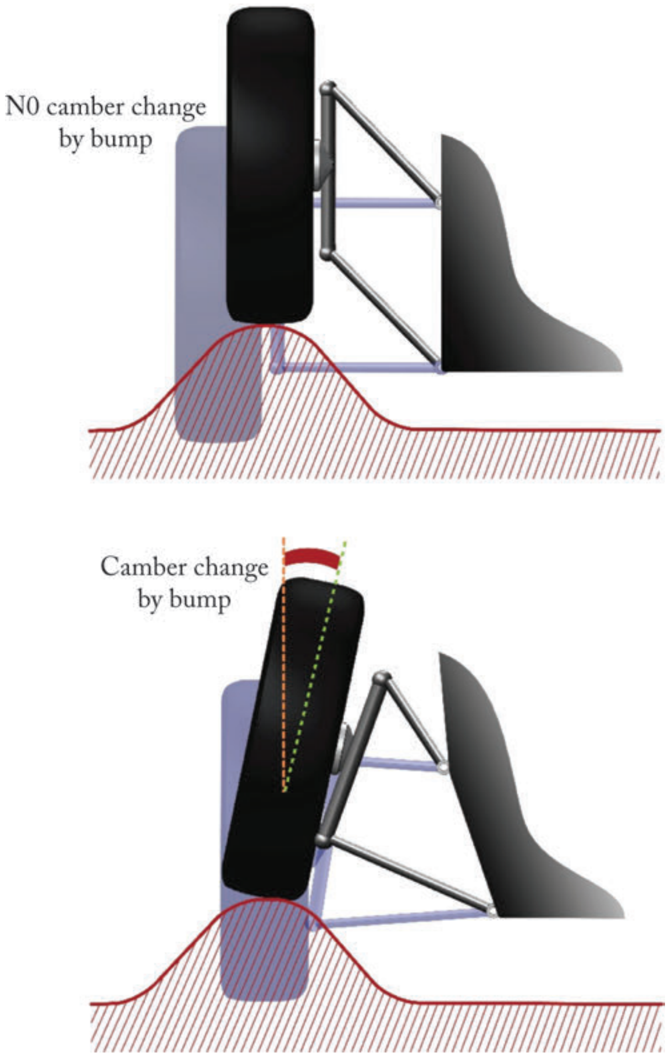


Figure 1.4: Effects of suspension design on camber change due to road bump.

- **Good space efficiency:** It comes as no surprise that the space utilized by a suspension system may create difficulties for the installation of other components of the vehicle. Under the hood, the suspension system should leave enough room for an engine and other components. Also, the suspension of the rear axle should not interfere with the vehicle trunk and, instead, occupy only its internal space (Figure 1.5).

- **Good structural efficiency:** The suspension system should be able to handle the vehicle's weight and all applied forces and moments in the contact area between the wheel and the road. The suspension mechanism must feed loads into the body in a well distributed manner and prevent the transfer of concentrated forces onto the vehicle's body (Figure 1.6).

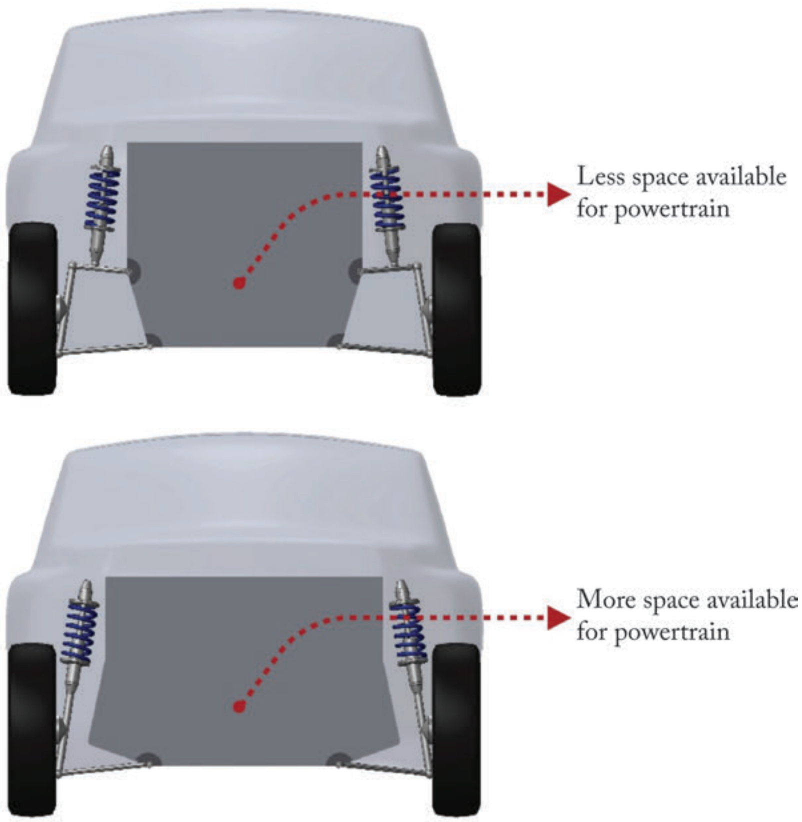


Figure 1.5: Space efficiency of suspension systems.

- **Good isolation:** Improving ride quality and isolating road roughness is one of the most important tasks of a suspension system.
- **Low weight:** Due to the road irregularities, the kinematic energy of a suspension system is proportional to its mass. Higher kinematic energy results in stronger transmitted shocks to the vehicle body. This effect clearly decreases the ride quality. To minimize this negative effect, we should minimize the suspension mass by using optimized designs and/or lightweight material. Lightweight materials may increase the cost and, therefore, a balanced design is needed for any suspension system.

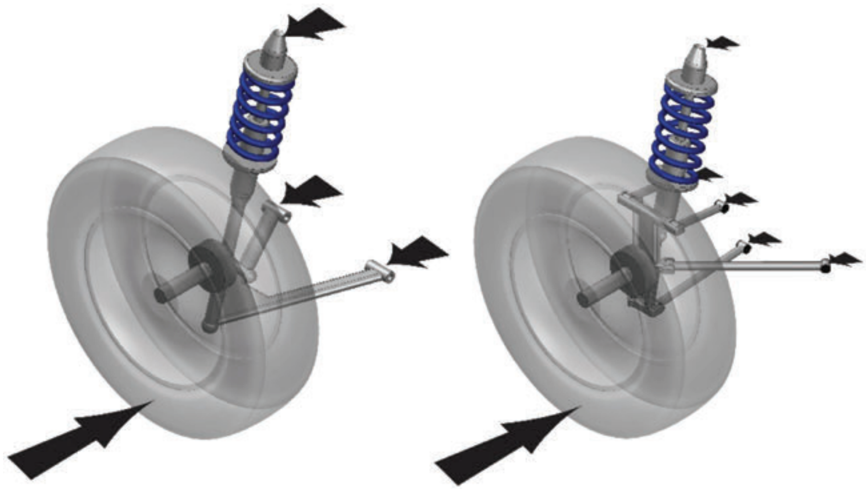


Figure 1.6: Structural efficiency in suspension systems

- **Long life:** No one enjoys having to repair their car frequently, so the suspension must be as durable as any other part of a car. A durable system is able to resist wear, pressure, or damage, all of which play important roles in the success of a product.
- **Low cost:** While defining a low enough cost is a subjective matter, the suspension as a vehicle sub-system should be affordable. High performance suspension systems are more expensive, and mainly used in premium vehicles. Using a high number of bushings and lightweight materials certainly improves the ride quality, noise isolation, and performance of the system, but they also increase the cost of the product.
- **Others:** Other suspension features may include anti-dives and anti-squats. When a vehicle is breaking, a dive occurs where the front of the vehicle dips and the tail rises. A similar but opposite action, a squat, happens during acceleration. This rotational movement is slight but since the human body is very sensitive to pitch motion, mitigating for this movement in the passenger cabin allows for greater ride quality.

CHAPTER 2

Classification of Suspension Systems

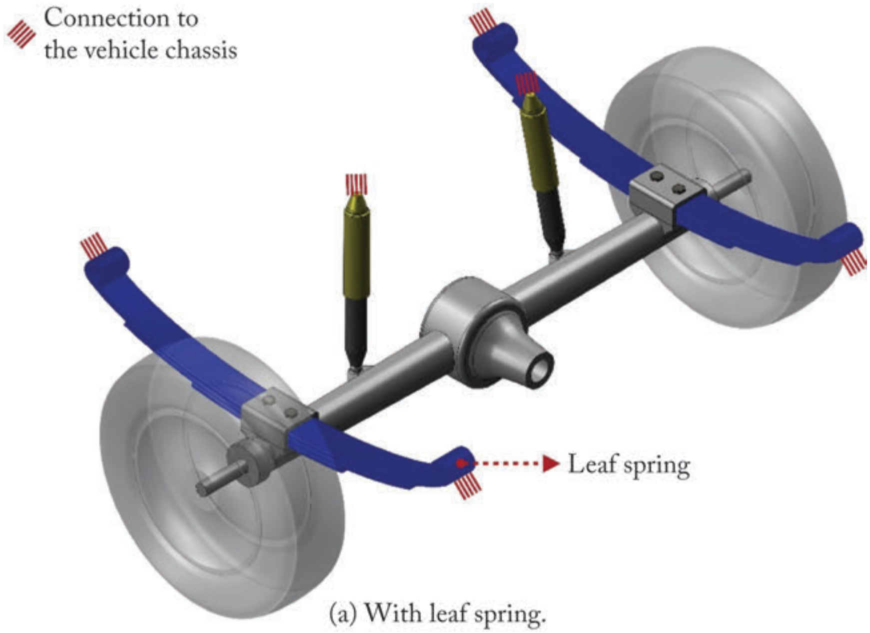
2.1 DIFFERENT SUSPENSION TYPES

2.1.1 SOLID AXLE

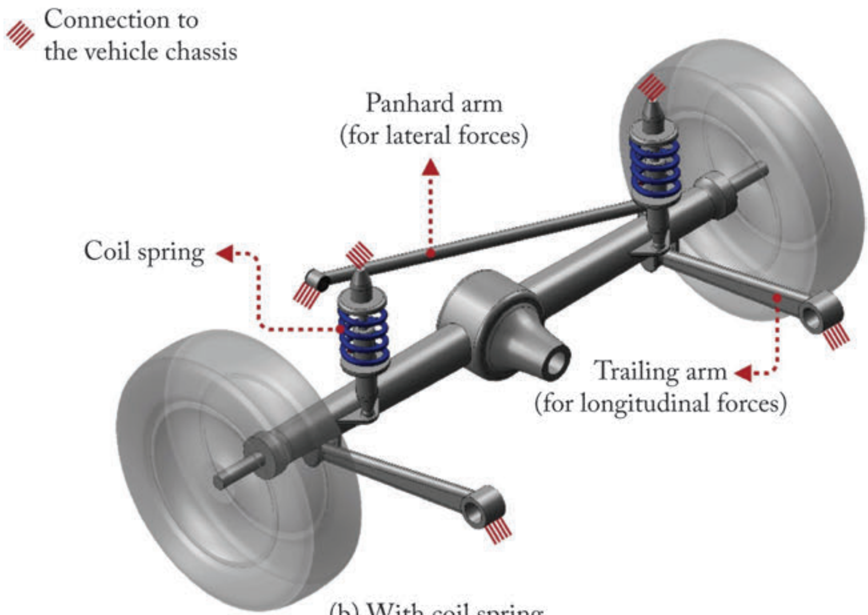
A solid axle is known to be the first suspension type used in vehicles where the wheels are mounted at either ends of a rigid beam (Figure 2.1). A solid axle system generally uses leaf springs, however, there are some equipped with coil springs. A solid axle suspension system is not classified as an independent type of suspension, so any movement of one wheel is transmitted to the other wheel. Solid axles have some advantages in their load carrying capacity; solid axles are also affordable and durable. However, they have many disadvantages when considering passenger cars. Leaf springs result in unwanted transmitted noise and vibration. Rigid beams considerably reduce ride quality, and their size and dimensions demand enough space above the axle to accommodate the springs. Solid axle suspensions are commonly used in commercial vehicles where a high load carrying capacity is required. Table 2.1 presents a summary of details regarding the solid axle.

Table 2.1: Advantages and disadvantages of a solid axle suspension

Features	Description	Verdict
Independency	Mutual wheel influence	Bad
Camber angle	Fixed wheels to the axle	Bad
Roll angle	High roll center height	Moderate
Space efficiency	Need more space	Bad
Structural efficiency	Concentrated forces at mounting points	Bad
Isolation	Noisy due to leaf springs motion against each other	Bad
Weight	Heavy axle	Bad
Lifespan	Less moving components and maintenance	Good
Cost	Less components and no complex parts	Good



(a) With leaf spring.



(b) With coil spring.

Figure 2.1: Different types of solid axle suspension.

2.1.2 TORSION BEAM

The torsion beam is known as the most popular rear axle suspension system for small- and medium-sized front-wheel drive vehicles. A torsion beam suspension consists of two trailing arms with a flexible cross member in between, shown in Figure 2.2. The cross member absorbs all forces and moments and simultaneously works as an anti-roll bar.

The torsion beam is a semi-independent suspension. It boasts few components and a simple structure; therefore, it is light, cheap, easily assembled, and requires little space. The lightweight nature of the system results in an improved ride comfort, but cross member deformation lends itself to the tendency of lateral force oversteering and poor handling. Furthermore, cross member flexibility diminishes camber and body roll control. A summary of torsion beam axles is presented in Table 2.2.

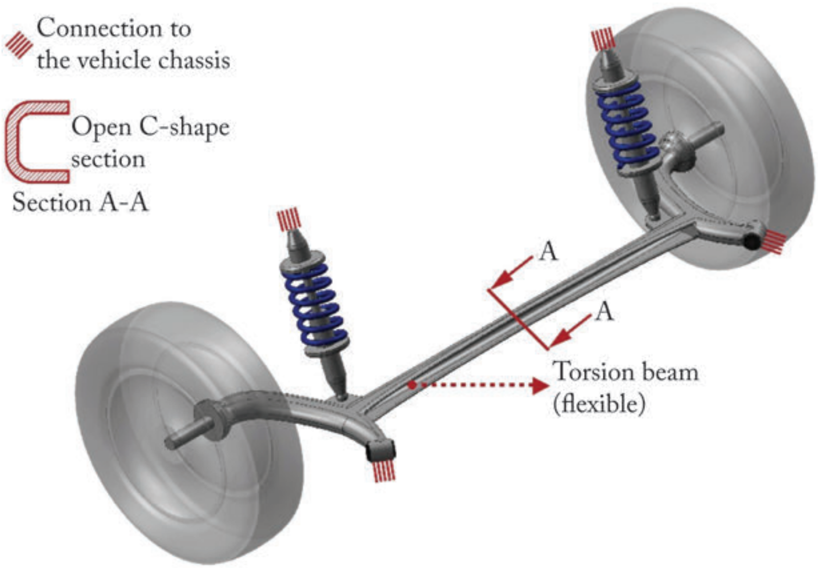


Figure 2.2: Torsion beam suspension.

Table 2.2: Advantages and disadvantages of a torsion beam suspension

Features	Description	Verdict
Independency	Semi- independent	Moderate
Camber angle	More flexibility due to cross member	Moderate
Roll angle	More flexibility due to cross member	Moderate
Space efficiency	Needs smaller space	Good
Structural efficiency	Concentrated forces in few mounting points	Bad
Isolation	Better isolation and no leaf spring noise	Moderate
Weight	Simple systems with light components	Good
Lifespan	Less moving components and maintenance	Good
Cost	Less components and no complex parts	Good

2.1.3 TRAILING ARM

The trailing arm suspension is used in the rear axle with either front or rear wheel drive vehicles, and it is classified as an independent suspension system. This suspension system consists of a control arm that is longitudinally connected to the body, while the wheels remain parallel to the body, so no camber and toe angle changes are caused by the wheel's vertical movement. However, during a body roll, the wheel camber changes significantly. To reduce this unwanted effect, the design of the trailing arm has been modified to a semi-trailing arm suspension, where the revolt axis of the control arm makes an angle with the vehicle's longitudinal axis, as shown in Figure 2.3. In a well-designed, semi-trailing, arm suspension, minor camber and toe angle changes happen either due to the wheel's vertical movement or the body roll. Although trailing and semi-trailing suspension systems provide good ride comfort, durability, and handling performance, they are now out-of-date because there are more effective independent suspension systems for vehicles. Figure 2.3 shows the trailing and semi-trailing arm suspensions. Their main features are listed in Table 2.3.

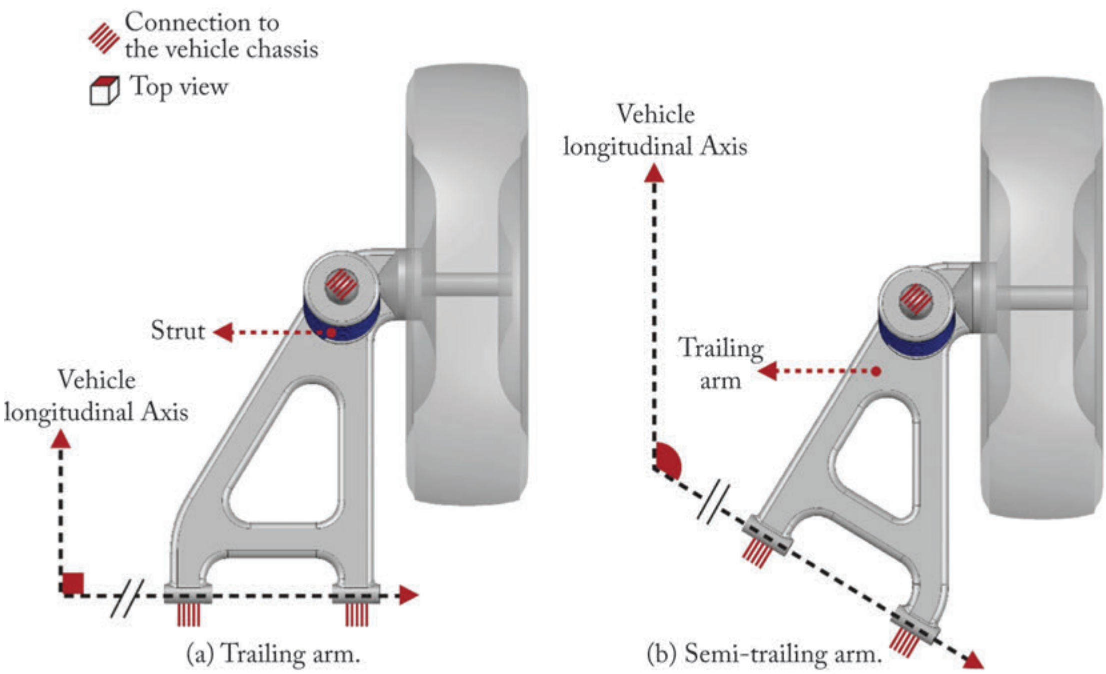


Figure 2.3: Trailing and semi-trailing arm suspension systems.

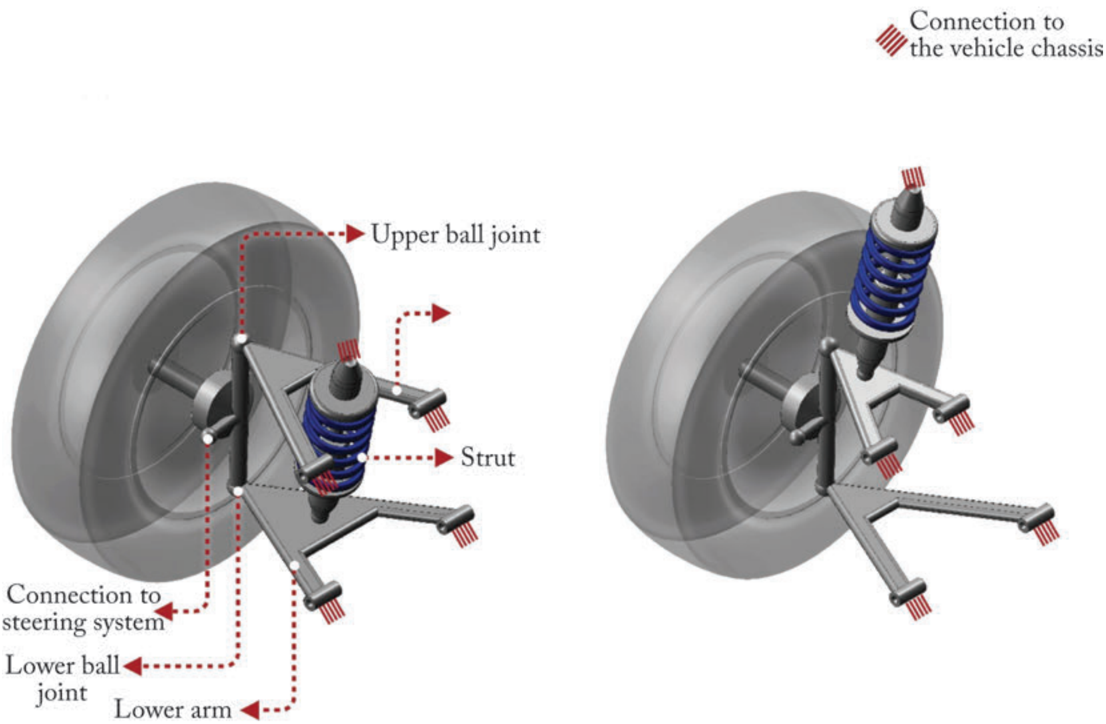
Table 2.3: Advantages and disadvantages of the trailing/semi-trailing arm suspension

Features	Description	Verdict
Independency	Fully independent	Excellent
Camber angle	By optimum design of the revolt axis	Good
Roll angle	By optimum design of the revolt axis	Good
Space efficiency	Needs smaller space	Good
Structural efficiency	Two connection points per wheel	Moderate
Isolation	Bushings are used at the connection points	Good
Weight	Dependent on the control arm design	Good
Lifespan	No complex elements	Good
Cost	Needs four bushings per axle	Moderate

2.1.4 DOUBLE WISHBONE

The double wishbone suspension is used in front and rear axles, and it consists of two control arms to hold the wheel as shown in Figure 2.4. The main advantage of the double wishbone suspension is its customizable kinematic design. The positions, lengths, and angles of the control arms specify the roll and pitch behavior of the vehicle (through the roll center height and the pitch pole position), and the double wishbone can influence wheel angle changes after bump-related movement. The designer can adjust the suspension characteristics for the particular needs of different vehicle types to achieve the best handling and stability performances.

The double wishbone suspension is large laterally, so it minimizes the undesirable effects on the ride quality. This type of suspensions is relatively expensive due to their complexity and number of joints. Connecting the spring to the lower wishbone as shown in Figure 2.4a, or to the upper wishbone, as shown in Figure 2.4b, may cause interference with the driving shaft or extra space for the suspension packaging, respectively. A summary of the features of double wishbone suspensions is provided in Table 2.4.



(a) Spring on lower arm (b) Spring on upper arm

Figure 2.4: Different types of double wishbone suspension.

Table 2.4: Advantages and disadvantages of a double wishbone suspension system

Features	Description	Verdict
Independency	Fully independent	Excellent
Camber angle	Customizable	Good
Roll angle	Customizable	Good
Space efficiency	Needs larger space, especially in the lateral direction	Bad
Structural efficiency	Four connection points per wheel	Good
Isolation	Bushings are used at the connection points	Good
Weight	Dependent on wishbone design and materials	Moderate
Lifespan	No complex elements	Good
Cost	Could be expensive	Moderate

2.1.5 MACPHERSON STRUT

The Macpherson strut is usually used as the front axle of small- and medium-sized front-wheel drive vehicles. The upper control arm in the double wishbone is replaced by a strut that is rigidly connected to the wheel's spindle (Figure 2.5). The lower control arm mainly bears lateral and longitudinal forces, and its structure and shape play important roles in wheel compliance control. This independent suspension is small enough to allow for an under-the-hood installation of the engine following the installation of the suspension. It is cheaper and lighter than the double wishbone suspension; however, replacing the upper arm with the strut decreases the roll, pitch, and wheel angle control, so from a vehicle handling point of view, the Macpherson strut is merely acceptable. A summary of the features of double wishbone suspensions is provided in Table 2.5.

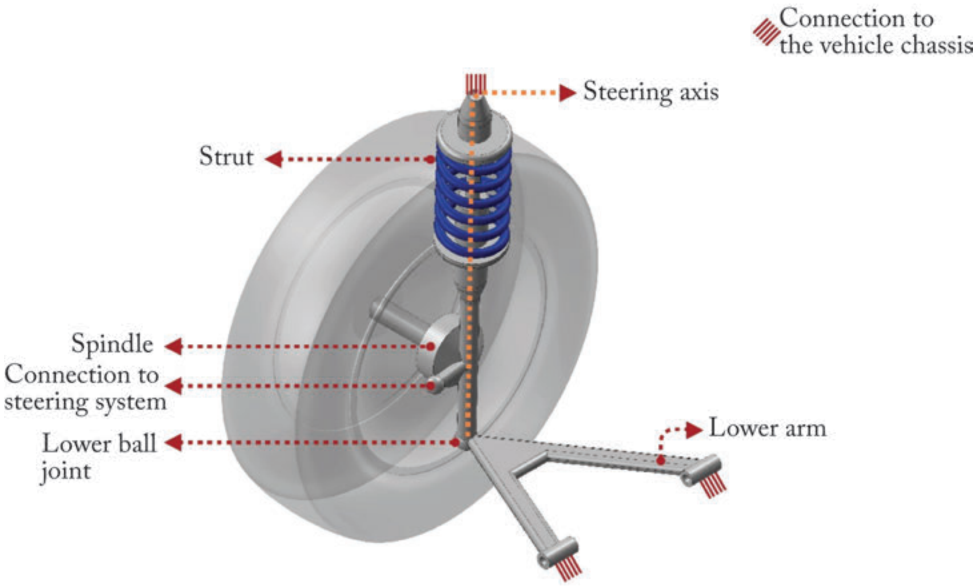


Figure 2.5: Macpherson strut suspension system.

Table 2.5: Advantages and disadvantages of a Macpherson strut suspension

Features	Description	Verdict
Independency	Fully independent	Excellent
Camber angle	Not as good as double wishbone	Moderate
Roll angle	Not as good as double wishbone	Moderate
Space efficiency	Needs smaller space	Good
Structural efficiency	Concentrated forces on three connection points	Moderate
Isolation	Use of bushings at the connection points	Good
Weight	Lightweight	Good
Lifespan	Strut has a more complex design than other suspensions	Moderate
Cost	Simpler design compared to double wishbone	Good

2.1.6 MULTI-LINK

In recent years, multi-link suspension systems have become popular in the rear axle of luxury vehicles. As shown in Figure 2.6, a multi-link suspension is formed with five or more links to make a truss structure, where most of the links are just under the axial load. There are many design parameters in this configuration, so multi-link suspensions can be designed to better control the wheel angles and body movements. Also, the truss structure of a multi-link suspension makes it very light. The high number of links and joints make multi-link suspensions complicated and expensive with more maintenance costs. A summary of the features of multi-link suspensions is provided in Table 2.6.

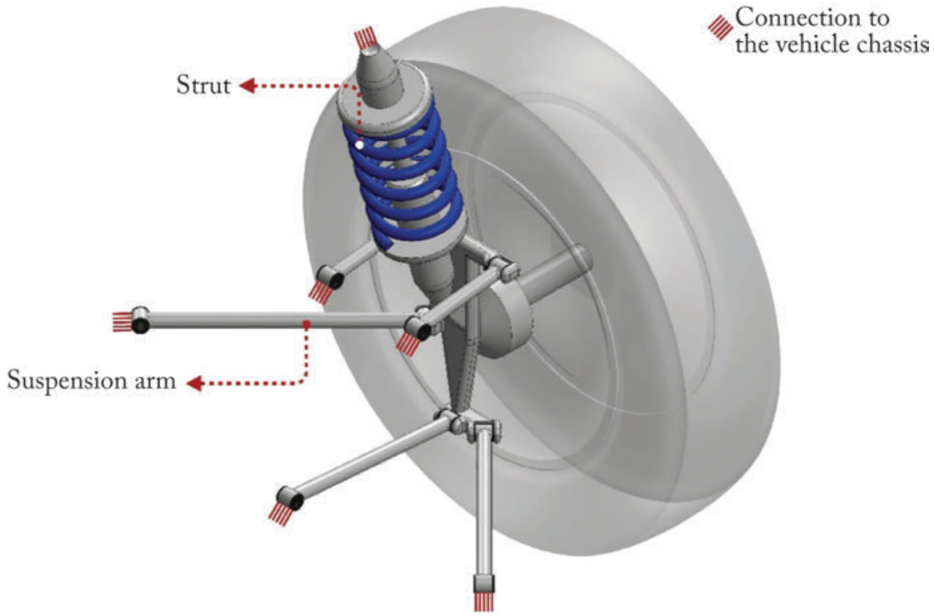


Figure 2.6: Multi-link suspension system.

Table 2.6: Advantages and disadvantages of a multi-link suspension system

Features	Description	Verdict
Independency	Fully independent	Excellent
Camber angle	Can be optimized through the design	Excellent
Roll angle	Can be optimized through the design	Excellent
Space efficiency	Can be designed for tight spaces	Moderate
Structural efficiency	Several connection points	Excellent
Isolation	Use of bushings at the connection points	Good
Weight	Truss structure	Excellent
Lifespan	Needs extra maintenance due to complexity	Moderate
Cost	Expensive	Bad

Example 1: Select the appropriate suspension system type in front and rear axles of each of the following vehicles:

a. A compact sedan

b. A mid-size SUV

c. A commercial van

d. A full-size luxury sedan

Answers:

a. Compact sedans are considered to be affordable vehicles, and a space efficient suspension system is crucial due to their compact nature. Therefore, an appropriate suspension system should be relatively small and inexpensive. As such, the most common configuration in the compact sedan is the Macpherson strut system for the front axle, and the torsion beam system for the rear axle. This configuration provides a moderate ride quality and handling at a cost-effective range.

b. A mid-size SUV may sometimes drive off-road, where significant forces act on the suspension system. Thus, durability, structural efficiency, and isolation should be prioritized. From a safety perspective, body roll control is paramount since SUVs are susceptible to roll over due to their high height. Therefore, for a mid-size SUV, an improved Macpherson strut or a double wishbone system should be implemented in the front axle, and a multi-link system should be chosen for the rear axle. The shape and number of bushings of the Macpherson strut are improved when designed for an SUV due to greater load bearing capacity.

c. A commercial van is usually used to distribute goods in cities, so the suspension system in rear axle must be able to handle large loads every day. Durability and a low maintenance cost is very important in this type of vehicle. An improved Macpherson strut or a double wishbone system is used in the front axle, and a solid axle is suitable for the rear axle.

d. Consumers purchasing a full-size luxury sedan expect the very best in terms of vehicle features and performances. The suspension system should provide excellent handling through accurate camber and roll control, and superb ride comfort quality through perfect vibration isolation. The double wishbone or multi-link system can be used in the front axle of a full-size luxury sedan, and a multi-link can be used in the rear axle.

2.2 SPRING TYPES

A spring is an elastic object used to convert kinematic energy into potential energy and vice versa. The spring is an important component in a suspension system as it bears the normal forces and crucial to vehicle ride quality and handling. Therefore, different types of springs were invented in order to improve various suspension performances.

2.2.1 COIL SPRINGS

Coil springs appeared in the early 15th century. Coil springs are made of elastic materials shaped into a helix. The traditional coil spring as shown in Figure 2.7a obeys Hooke's law and has a constant spring rate; however, in modern cars, it should obey a variable spring rate since the spring provides the vehicle with a soft ride when traveling with light loads and a firmer ride when traveling with heavier loads. Also, this variable spring rate provides improved body and angle control. There are multiple methods of creating the variable rate coil spring as described below and shown in Figure 2.7b:

- variable diameter winding of the elastic material which decreases toward the ends of spring;
- variable bar diameter where the bar tapers nearing each end;
- an extended elastomer bump stop; and
- donical or beehive shape winding of the elastic material.

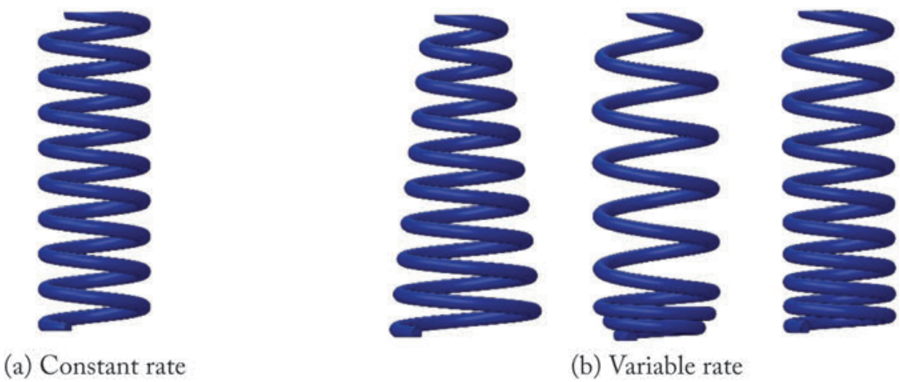


Figure 2.7: Coil springs with (a) constant rate and (b) variable rate.

2.2.2 TORSION BARS

A torsion bar is a torsional spring used as the main spring in a suspension system. It is constructed as a long bar whose one end is connected to the vehicle chassis and its other end is joined to the suspension control arm, as shown in Figure 2.8. The vertical motion of the wheel results in a rotational movement in the bar around its axis, which is resisted by the bar's torsional stiffness. Rear suspension torsion bars are mainly used in small European cars, whereas some sport utility vehicles and trucks have them in their front suspensions.

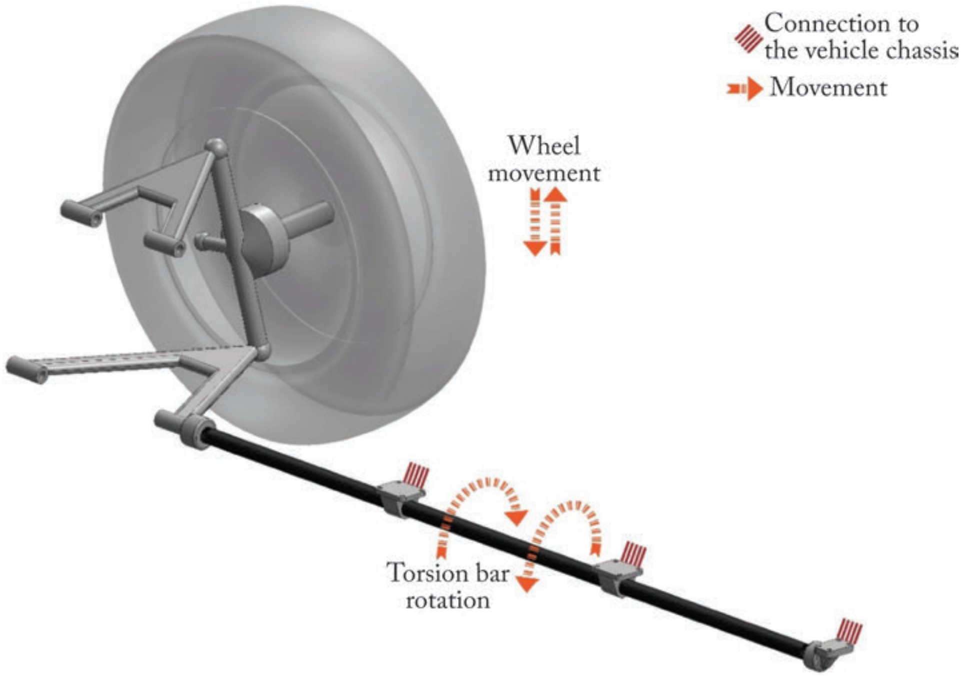


Figure 2.8: Torsion bar.

2.2.3 LEAF SPRINGS:

Leaf springs consist of a number of arc-shaped longitudinal strips, as shown in Figure 2.9, where the center of the arc is supported by axle. Usually, one end of the leaf spring is attached directly to the frame and the other end is connected through a shackle link. Leaf springs are mainly used on solid rear axles, which primarily appear in trucks.

Figure 2.9: Leaf spring.

2.2.4 ALTERNATIVE SPRINGS

Alternative springs are used in advanced suspension systems such as the self-leveling and hydro-pneumatic systems. The main advantage of this kind of spring is the independency of suspension characteristics from carrying loads. Alternative springs include the air spring and the gas spring.

- **Air spring:** An air spring, shown in Figure 2.10, is a pillow that inflates via an electric air pump. It provides high quality ride comfort without dependency on carrying loads. The cost of an air spring is high and they are mainly used in luxury sedans and heavy vehicles like buses and trucks. Air springs provide a self-leveling feature that is useful in many vehicles.

Figure 2.10: Air spring.

- **Gas spring:** The gas spring is a piston-cylinder filled with compressed gas. The strut shape of the gas spring makes it very practical for use in a variety of suspension systems. Gas springs can also be used as a damper; as such, it can be called a "slow-dampened spring," and it can be designed for parts that are operated slowly like heavy doors and windows. It can also be used in objects that are operated quickly, where they are termed as a "quick gas spring."

2.2.5 ADVANCED MATERIALS SPRINGS

Composite materials have the characteristics that can replace metal springs. Composite springs are lightweight, durable, and highly corrosion resistant. However, their difficult manufacturing process and high cost prevent them from mass implementation in all vehicle classes.

2.3 SHOCK ABSORBER TYPES

A shock absorber has a piston that moves inside a sealed, oil-filled cylinder. There are one-way valves on the piston that allow the oil to slowly flow from one chamber to another. In a running vehicle, shock absorbers dampen vibrations to control vehicle handling and ride comfort. Usually, shock absorbers convert kinematic energy to heat. Three types of shock absorbers are described below.

2.3.1 TWIN TUBE

The twin tube shock absorber consists of two cylindrical tubes, as shown in Figure 2.11a. At the bottom of the inner tube, there is a foot valve that allows oil to flow between the tubes. The piston moves up and down into the inner tube while the outer tube serves as reservoir. Oil moves between upper and lower chambers of the inner tube via the piston valve as well as the inner and outer tubes via the foot valve. The orifice valves in the piston control most of the damping with assistance from the foot valve.

2.3.2 MONO TUBE

As its name indicates, a mono tube shock absorber consists of one tube that is divided into sections by two pistons, the working piston and the floating piston, shown in Figure 2.11b. The gas and oil are separated by the floating piston, which plays a role analogous to that of the foot valve in twin tube shock absorbers. Similar to that of the twin tube, the orifices in the piston valve produce most of the damping, and during more aggressive movements, the floating piston pushes farther into the gas chamber to quickly increase gas pressure and provide an additional damping force. The length of the cylinder and piston in a mono tube is consistently larger than those in the twin tube since they require a larger area to operate.

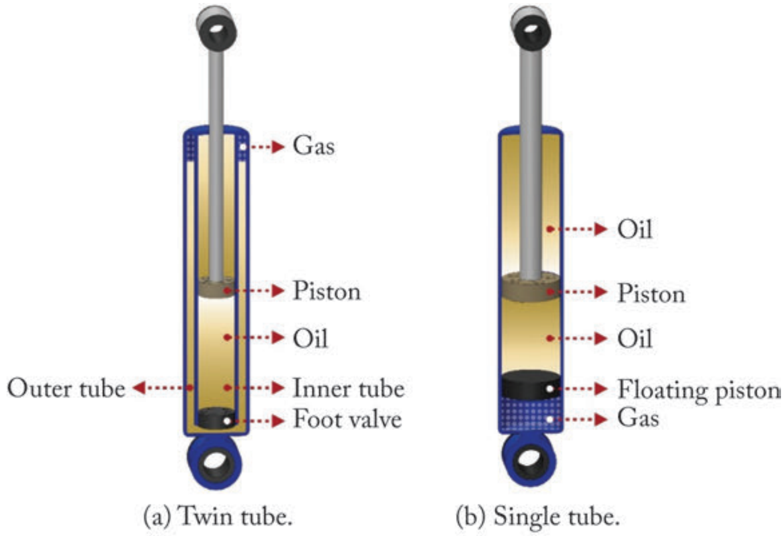


Figure 2.11: Shock absorbers: (a) twin tube and (b) mono tube.

2.3.3 ADJUSTABLE AND ADAPTIVE

Ideally, different loads and driving situations require different suspension stiffness and damping. As such, adjustable shock absorbers are introduced within suspension systems. Lower damping results in an improved high-frequency ride comfort, whereas the improvement in handling and low-frequency ride comfort are the results of higher damping.

The first generation of adjustable shock absorbers was manual, and they appeared on luxury vehicles in the 1950's. The second generation was introduced in the mid-1980's, and they used a solenoid valve to control the hydraulic circuit. They were able to automatically adjust the damping at three or four levels. The third generation in the 1990's used variable valves to continually change the damping value. In the 2000's, magneto rheological dampers were introduced as the fourth generation, where the cylinder is filled with a mixture of magnetized iron particles in a synthetic hydrocarbon oil. A coil is used around the cylinder body while a stake of permanent magnets is placed in the piston, as shown in Figure 2.12. The electromagnetic filed are able to continuously change the viscosity of the magneto rheological fluid. When it is off, the fluid flows through the piston passage freely, whereas, when the magnetic field increases, the fluid becomes more viscous, which changes the damping force.

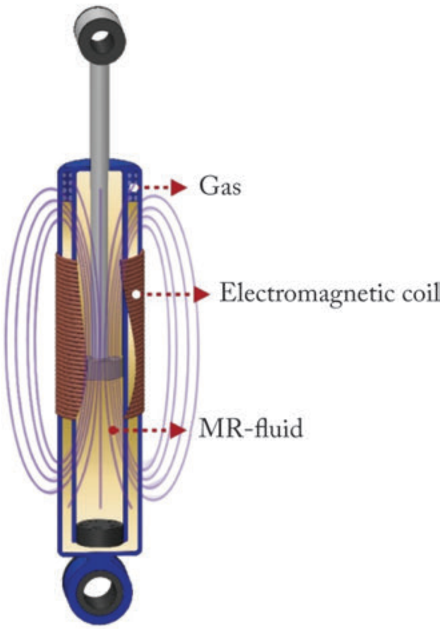


Figure 2.12: Magneto rheological shock absorber.

2.4 ANTI-ROLL BAR

As shown in Figure 2.13, to reduce body roll during cornering, an anti-roll bar connects the two opposite wheels on an axle through a torsional bar that acts as a torsional spring. When both wheels of an axle move simultaneously in the same direction, the anti-roll bar remains inactive, but if the wheels move in opposite directions, the bar is subjected to torsion and is forced to twist. The anti-roll bar decreases body roll by increasing roll stiffness without affecting the main suspension stiffness.

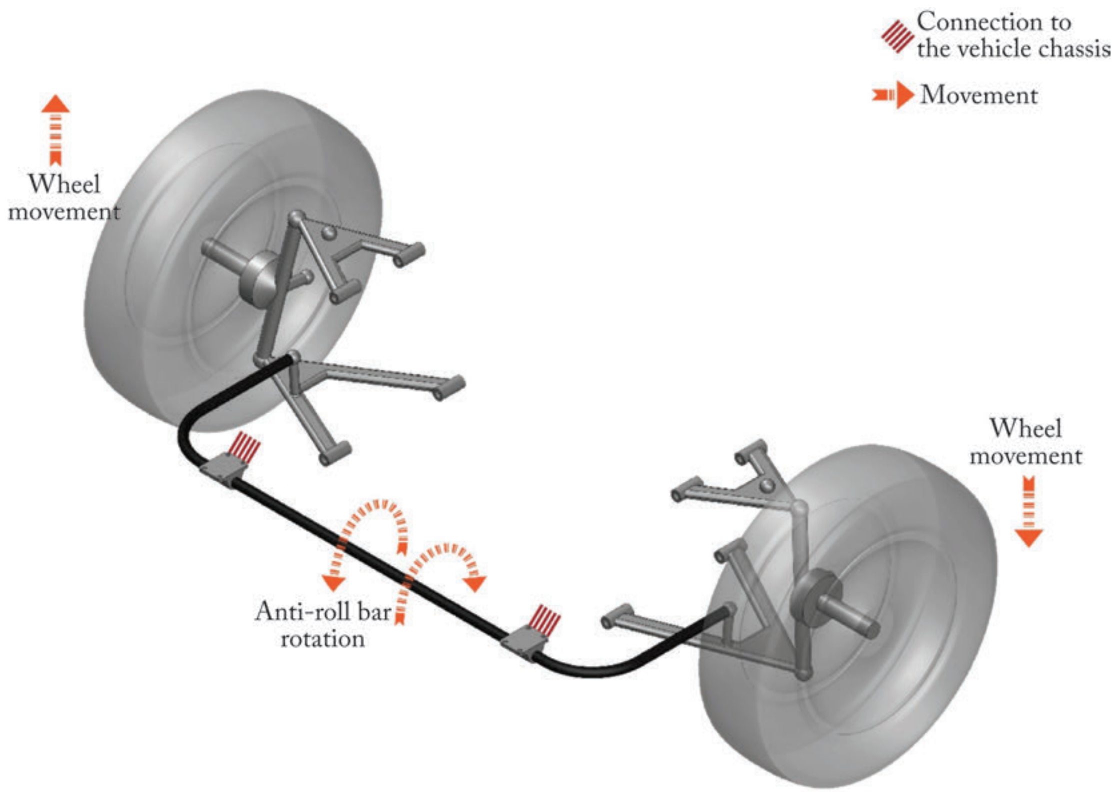


Figure 2.13: Anti-roll bar connects the wheels of an axle to provide torsional stiffness.

CHAPTER 3

Advanced Suspension Systems

3.1 SELF-LEVELING SUSPENSIONS

For vehicles with a high payload to curb weight ratio, when the car is heavy-laden, the car's ride height changes significantly that negatively affects ride comfort and ride handling. The Citroën introduced the first self-leveling rear suspension in 1954, which maintained a constant ride height for the vehicle.

Early self-leveling suspension systems were called air suspensions since vehicle height was changed by pumping air into flexible bellows. The modern system is a hydro-pneumatic suspension, whose main parts are shown in Figure 3.1. It contains a reservoir and a tank where oil flows via a pump. By pumping the oil in and out of the suspension chamber, the piston moves up or down, resulting in the vehicle body moving vertically. The hydro-pneumatic suspension is always fitted to the rear suspension of a car and the driver can adjust ride height manually. The system may operate slowly to adjust the vehicle's static ride height according to load, or it may operate fast enough to react to transient pitch movement. Some modern types of hydro-pneumatic suspensions can be classified in the semi-active category.

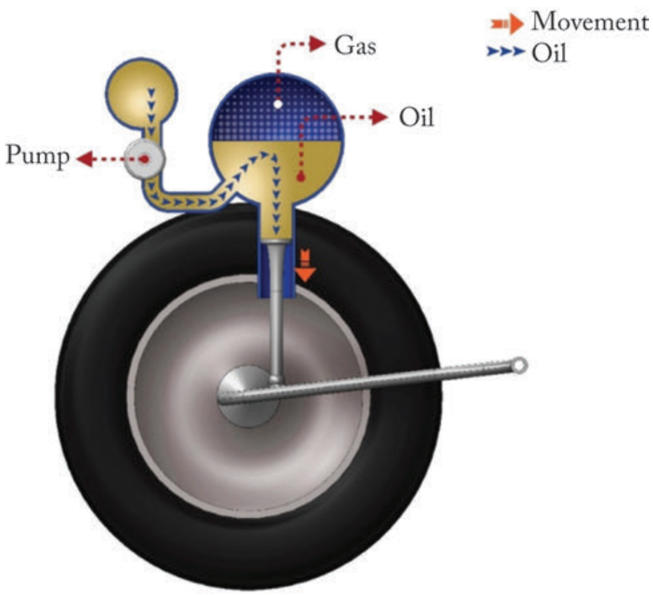


Figure 3.1: Hydro-pneumatic self-leveling suspension.

3.2 SEMI-ACTIVE SUSPENSIONS

The main working principle of a semi-active system is the suspension damping control that provides a distinct advantage in both ride quality and handling. This system includes adaptive shock absorbers as its main part, acceleration sensors to detect road conditions and vehicle movements, vehicle speed sensors, and an Electronic Control Unit (ECU) that controls the shock absorber. When driving in a straightaway, the ECU reduces the damping force to provide ride comfort; however, in cornering maneuvers, the damping force is increased to improve vehicle handling and safety. A semi-active suspension can only control low frequency (1-4 Hz) body motions (bump, pitch, or roll), whereas the high frequency range must already be handled by passive suspension parts. This system is widely used in up-market models. Figure 3.2 shows the main parts of a semi-active suspension.

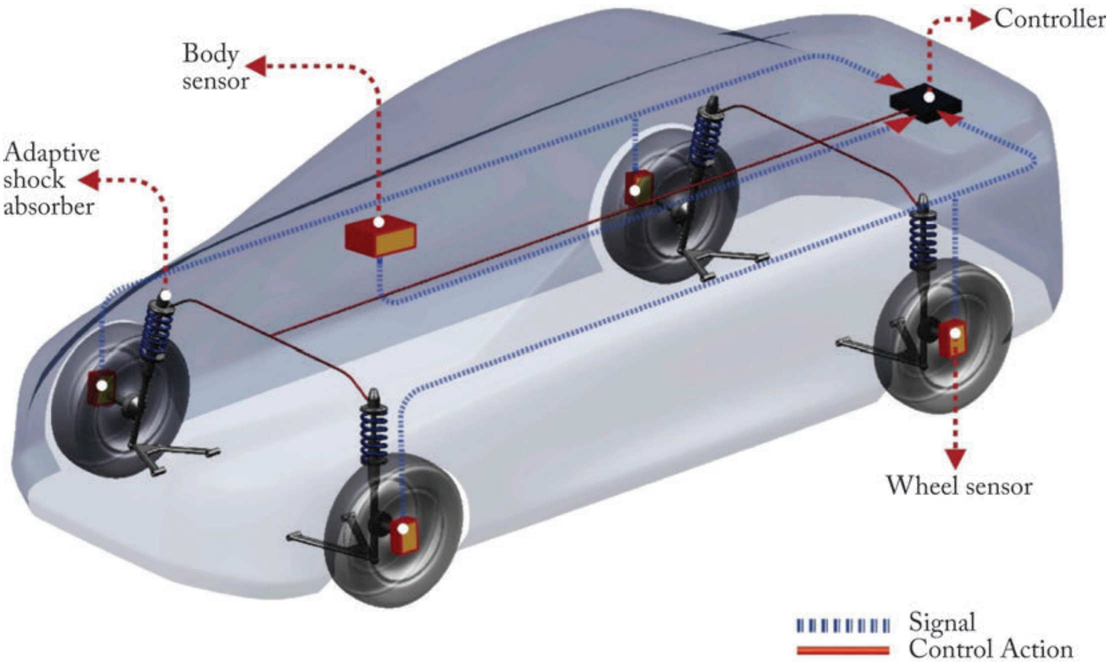


Figure 3.2: Semi-active suspension system.

3.3 ACTIVE SUSPENSIONS

Active suspensions are fundamentally different from traditional suspension systems. A traditional suspension stores energy in a spring and dissipates it via a shock absorber whose characteristics are fixed. An active suspension is also able to store and dissipate energy, but the damper-spring strut is replaced with a high-speed actuator. Active system configurations include an ECU and when a hydraulic actuator is used, a fast-response high-power hydraulic system that includes servo hydraulic actuators, a pump, servo valves, reservoirs, and sensors.

This system is able to handle bumps and rough roads much better by changing the suspension vertical forces in real time. Fully active systems can control body motions (pitch, roll, and bounce) as well as wheel hob motion over the wide range of frequencies (0–30 Hz). Its high cost, high power consumption, and complexity are the known disadvantages of full active suspensions. From a control strategy point of view, active systems could use preview control methods that can be divided into two main categories. The wheel base preview method that can detect wheel conditions, and the look ahead preview method that can predict future wheel conditions.

During the 1980's, few vehicles, mainly test vehicles and most notably from Lotus and Volvo, used the active suspension technology. During the 1990's, interest in active systems declined due to their cost, complication, and high power consumption. There are a few active systems on the market but they are not true, fully active systems since they are a combination of a passive system for the high frequency range and an active system for the low frequency range. Figure 3.3 shows the use of a fully active suspension system in a car.

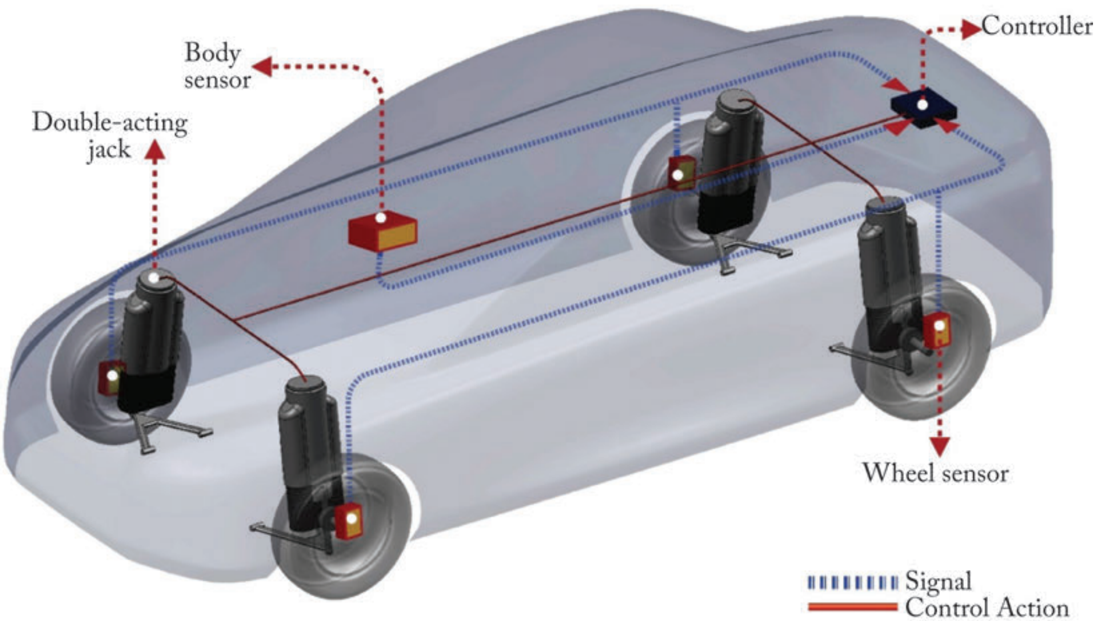


Figure 3.3: Active suspension system components.

CHAPTER 4

Analysis and Design of Suspension Mechanisms

4.1 SUSPENSION GEOMETRY

Suspension geometry plays an important role in wheel angles and movements, all of which affects vehicle road holding, handling, and safety. Suspension kinematic movements include wheel movements during vertical travel and steering as well as the movements due to the body roll and pitch. To study suspension kinematics, the SAE vehicle's coordinate system, XYZ, on the center of gravity of the vehicle and the wheel's coordinate system, xyz, are defined in Figure 4.1.

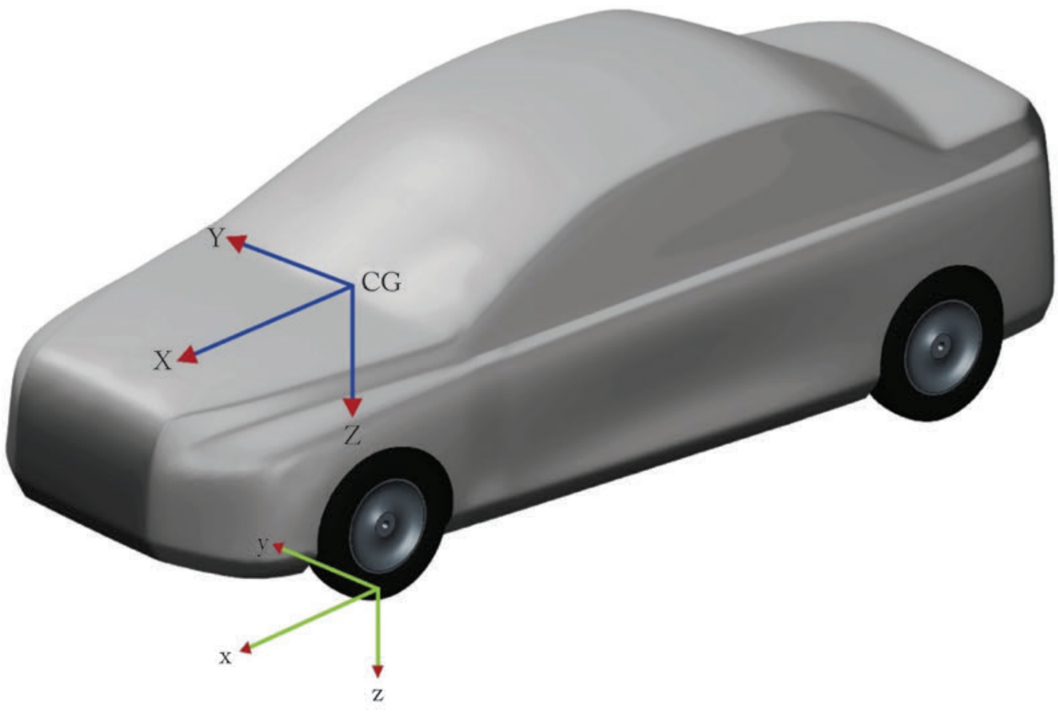


Figure 4.1: Vehicle and wheel coordinate systems.

4.1.1 ROLL CENTER AND ROLL AXIS

The roll center is a point on the y-z and X-Z planes where the lateral force applies on the vehicle body without kinematic roll occurring. The axis in the X-Z plane that goes through the front and rear roll centers is called the roll axis, and this is the axis that the body rolls around. The heights of the roll centers and roll axis play an important role in a vehicle's handling performance and ride quality. To determine the roll center, Arnold Kennedy's theorem is used. If three bodies have motion relative to each other, their instantaneous centers should lie in a straight line.

To find the roll center in a double wishbone suspension, only the directions of the control arms are important. The virtual center of rotation, P, is obtained by extending the upper and lower control arms. Instantaneously, P is linked with the center of the tire and extended to find the point where it intersects with the X-Z plane; this point is called the roll center. Figure 4.2 shows the roll center determination process in a double wishbone system.

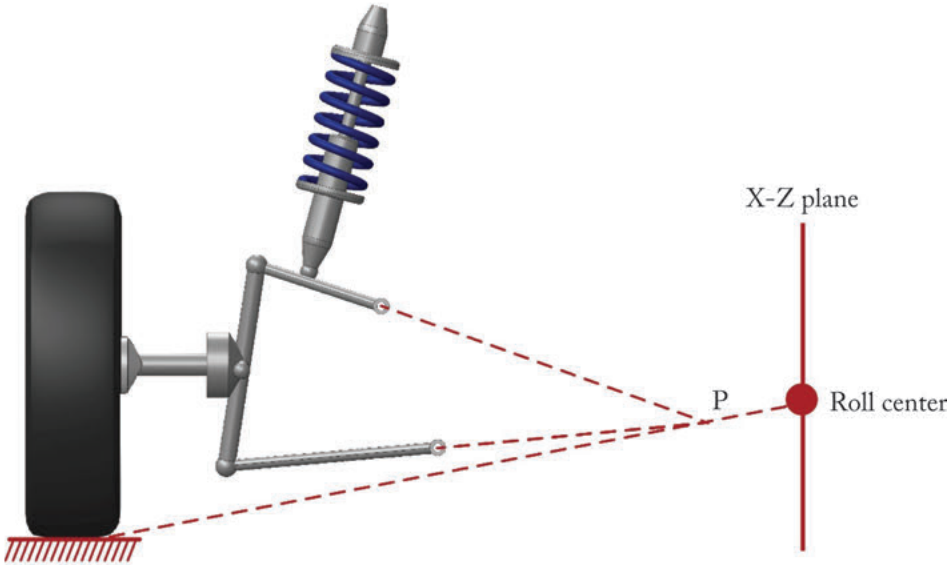


Figure 4.2: Body roll center for a double wishbone suspension.

In Macpherson suspensions, a vertical axis is created in the strut's end point where the intersection of it and the extended lower control arm provides a virtual center of rotation, P. Figure 4.3 shows the roll center determination process on Macpherson systems.

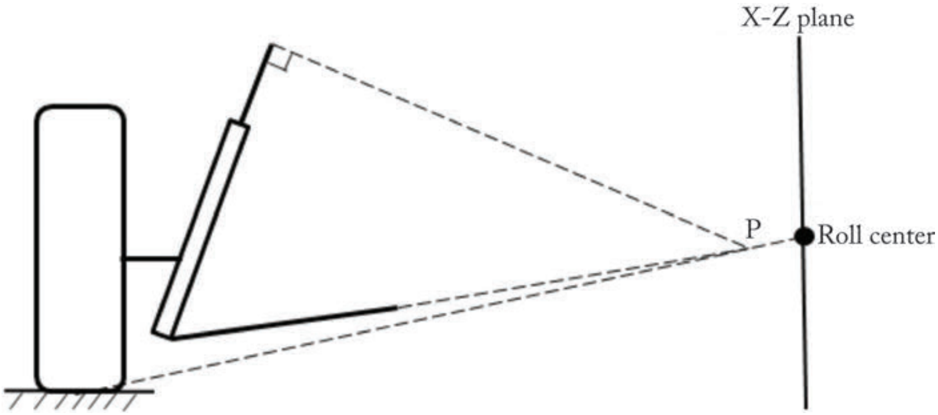


Figure 4.3: Body roll center for a Macpherson strut suspension.

In the torsion beam axle, whose top view is shown in Figure 4.4a, the bearing point is connected to the torsion beam center point, s, and intersects the Y-Z plane. From the rear view in Figure 4.4b, the virtual center of rotation, P, is linked with the center of the tire, and its intersection on the X-Z plane provides the roll center.

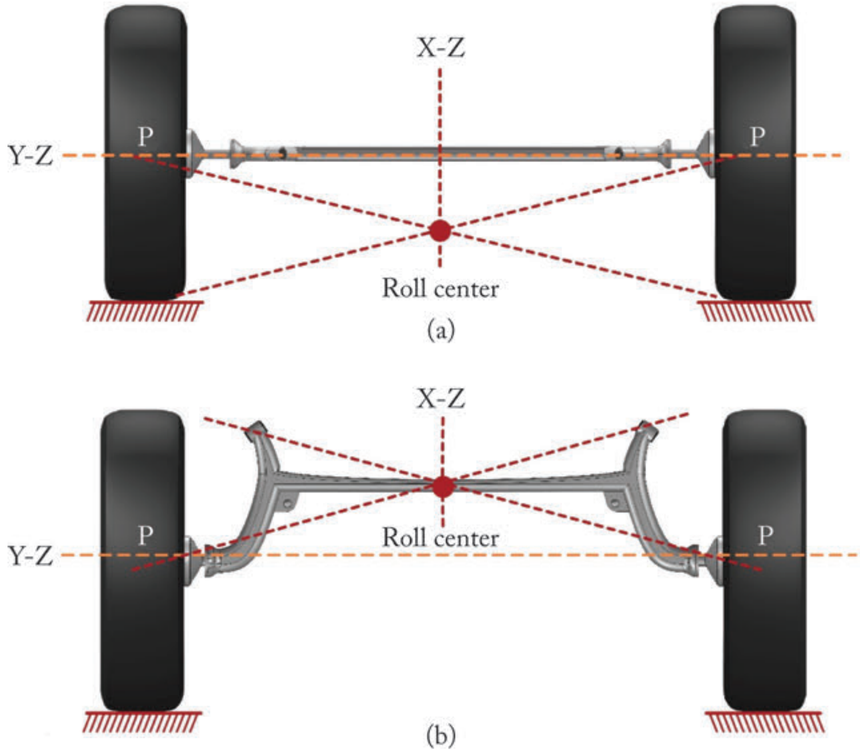


Figure 4.4: Body roll center in the torsion beam suspension (a) top view (b) rear view.

4.1.2 CAMBER ANGLE

The camber angle is the angle between the x-z plane of the wheel and a line perpendicular to the plane of road on the Y-Z plane of the vehicle. This angle is negative if the wheel is inclined inward, as shown in Figure 4.5a, and positive when inclined outward, as shown in Figure 4.5b. A positive camber would be useful for a reduction in tire wear due to the road profile. On the other hand, a negative camber provides better tire grip and improves handling.

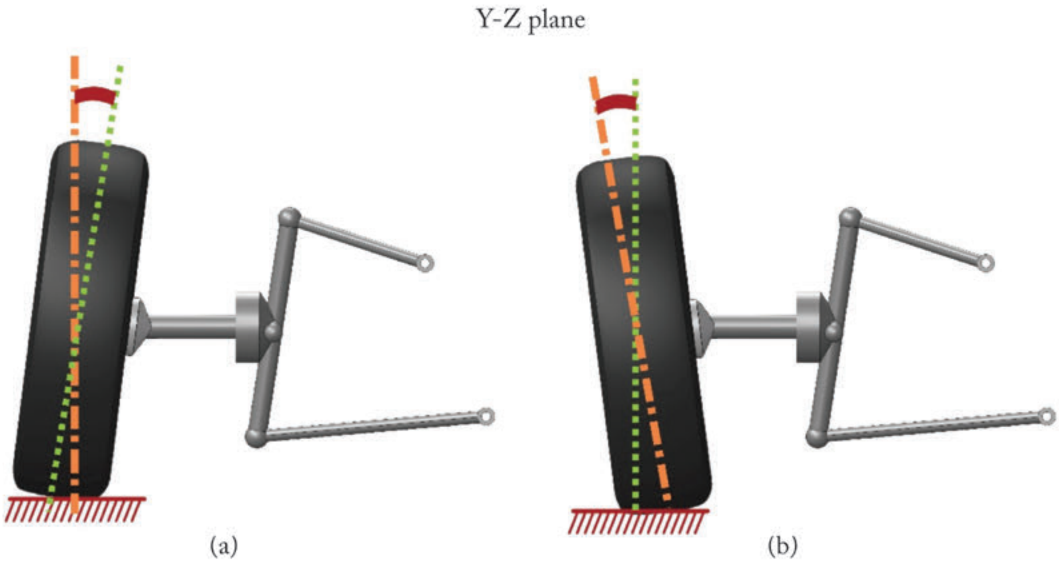


Figure 4.5: Camber angle: (a) negative camber and (b) positive angle.

The initial camber is set for a static wheel with factory settings where the rear wheels are usually more negative in camber angle than the front wheels. Wheel travel changes the camber angle, as shown in Figure 4.6. Proper suspension design should result in a negative camber during wheel travel bump and a positive camber during rebound. This design is useful during cornering because the inner wheel that travels during a bump supports bigger loads than the outer wheel in rebound.

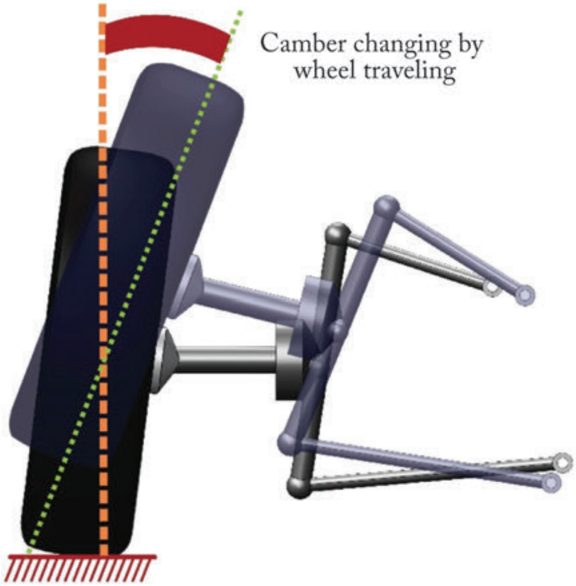


Figure 4.6: Camber changing by wheel travel.

4.1.3 TOE ANGLE

The toe is the angle between the x-z plane of the wheel and the X-Z plane of the vehicle. A "toe-in" occurs when the front part of wheel is turned inward, as shown in Figure 4.7, and a "toe-out" occurs when the wheel is turned outward. Rolling resistance and braking forces push the wheel slightly backward resulting in toe-out situations, and when traction pulls the wheel forward, it results in a toe-in situation. The toe angle should ideally be zero for the lowest amount of tire wear and rolling resistance. Therefore, manufacturers tend to define a toe angle on a static wheel that goes to zero under working conditions. Usually, to offset the braking forces on the front axle that normally results in toe-out situations, manufacturers set the front axle with toe-in angles. This is done because the vehicle may be unstable if the absolute value of the angle presents as a toe-out. A rear drive axle is set with a toe-out because of its traction force, and a rear non-drive axle adjusts to toe-in to compensate for rolling resistance.

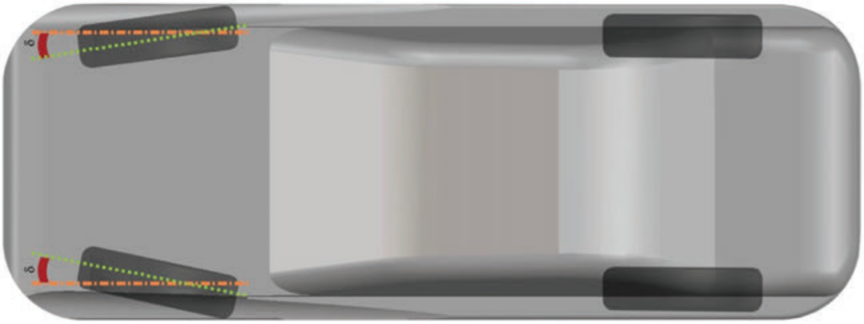


Figure 4.7: Toe-in in front wheels.

4.2 FUNDAMENTALS OF VIBRATION

Vibration theory is the field of science that studies the oscillatory motions of bodies and their associated forces. A vibrating system needs two elements: mass and elasticity; therefore, all un-rigid bodies are able to vibrate. The vibration system would have a shock absorber that turns kinematic vibrations into heat resulting the vibrations to diminish. There are two types of vibration: free vibration where there is no external forces to the system, and forced vibrations where at least one external force is applied to the body. The undamped time response of a body under free vibration is shown in Figure 4.8; the body repeats its oscillation about an equilibrium point at equal intervals of time.

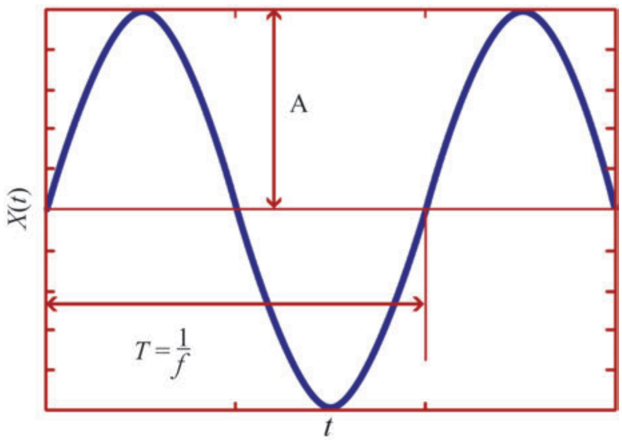


Figure 4.8: A simple sinusoidal vibration of a system under free vibration.

The equation of the motion of a system that vibrates freely can be written as:

$$x(t) = A \sin \omega t, \tag{1}$$

where A is the amplitude of oscillation, which is the maximum displacement of the oscillatory mass from the equilibrium point, and ω is the natural frequency that can be defined as:

$$\omega = \frac{2\pi}{T} = 2\pi f. \tag{2}$$

In Equation (2), T is the period of oscillation, which is the duration of time that a system needs to vibrate one cycle, and f is its reciprocal, the frequency. The relationship between period and frequency is:

$$T(\mathrm{sec}) = \frac{1}{\mathrm{f}(Hz)}. \tag{3}$$

Frequency is the number of cycles per unit time. Oscillations can usually be classified according to their frequency: the lower the oscillation frequency, the longer its wavelength and the lower its energy. A high-frequency oscillation has a shorter wavelength but higher energy.

4.2.1 VEHICLE NOISE, VIBRATION, AND HARSHNESS

A car has mass and elasticity, so according to the definition of vibration, it can have both free and force vibrations. There are a lot of rotational parts in a vehicle such as the engine, gearbox, wheels, differential and so on; each of them can generate forces that are applied to the vehicle's body. Not only do they have rotational motion, but they have also been installed asymmetrically within the vehicle, which makes vibrational situations worse. Moreover, regardless of the vehicle segment, a driver drives the car on different roads and road conditions—cars can be driven on-roads or off-roads. In addition, the contact between the tire and the ground, air flow over the vehicle body, engine, etc., different sounds that are also vibrations are generated. Such vibrations can considerably harm the vehicle ride experience.

On the basis of the aforementioned details, a car is exposed to a wide range of oscillations with different frequencies. The source of these vibrations can be internal or external. According to the vibration spectrum, we can classify them into three groups: Noise, Vibration, and Harshness (NVH). These classifications are based on the frequency of vibration, and each term refers to a specific frequency range that creates a variety of effects on the vehicle and passengers. These terms are defined as follows.

- **Vibration:** Oscillations with a frequency less than 25 Hz (frequency < 25 Hz) are known as vibrations that are associated with the oscillatory motion of the vehicle body; not only can this motion be seen, but it can also easily be felt by passengers. This sort of vibration is tangible, and passengers usually judge a vehicle's ride according to this range of vibrations. Thus, automotive engineers should pay close attention to control vibrations especially in this range. As mentioned in Chapter 3, one of the crucial tasks of a suspension system is to mitigate vibrations resulted from the contact of the tires and ground.

- **Harshness:** Oscillations with a frequency higher than 25 Hz and less than 100 Hz (25 Hz < frequency < 100 Hz) are called harshness. It cannot be seen, but passengers can feel the discomfort caused by this unwanted vibration. Harshness can be felt as shaking seats and dashboard; passengers cannot evaluate the harshness of a vehicle, but during long drives, they feel more tired after being in a vehicle with high harshness. Harshness can be generated by both external sources like road profile, and internal sources such as the engine, transmission, and brake.

- **Noise:** Oscillations with a frequency higher than 100 Hz (100 Hz < frequency) are classified as noise, and this refers to an unwanted sound. These oscillations cannot be seen nor felt, but they can be heard by passengers. Passengers can easily hear and be disturbed by unwanted sounds. Noise can be produced by internal components like the engine, transmission, and brakes, which are audible even at low speeds, or it can also be produced by external sources such as wind and tires, which can only be heard at high speeds.

Figure 4.9 shows the sources of vibration, harshness, and noise; moreover, it also shows the typical relationship between amplitude and frequency for each group. It is obvious that whenever the frequency increases, the amplitude decreases, and vice versa.

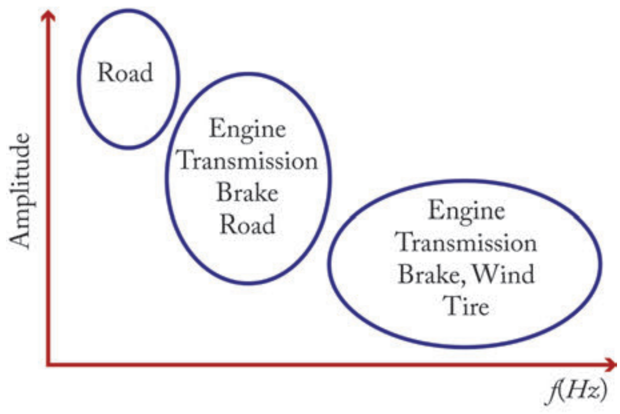


Figure 4.9: Different sources of vibration.

4.2.2 ROAD PROFILE

As mentioned, one of the crucial tasks of the suspension is to cope with the vibrations resulting from the contact between the tire and road. In order to design an effective suspension, we need to comprehensively know the road. Therefore, we will review the sinusoidal road profile, which is the simplest form of a road, and a study of the real road profile will follow. Finally, some typical road profiles to test the suspension will be introduced.

a. **Sinusoidal Road Profile**

The road's roughness is the principal source of vibration in a vehicle. In other words, road irregularities can induce oscillatory bounce, pitch, and roll motions on a sprung mass, as shown in Figure 4.10. The simplest form of road roughness is in a sinusoidal shape. The vibrations transmit directly onto the vehicle and negatively affect ride comfort, which can result in increased passenger and driver tiredness and discomfort. Moreover, a vehicle's unsprung mass or wheel is also excited by the road, which affects the tires' holding forces, and can eventually disturb handling, traction, and braking performance of the vehicle.

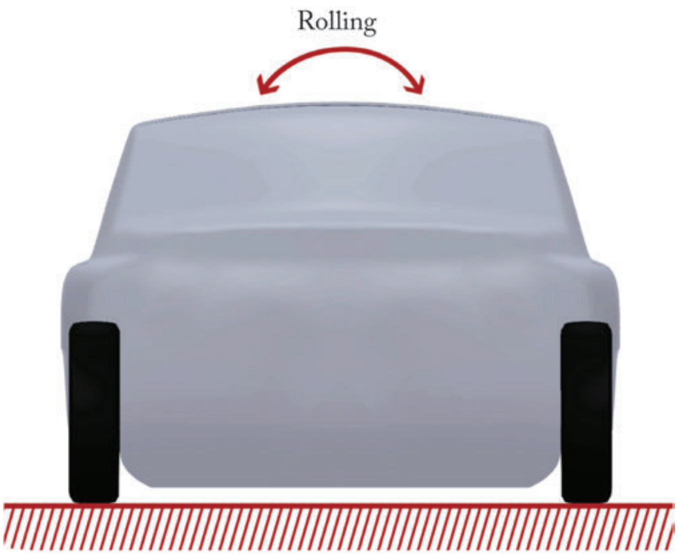


Figure 4.10: (a) Sprung mass movement—rolling.

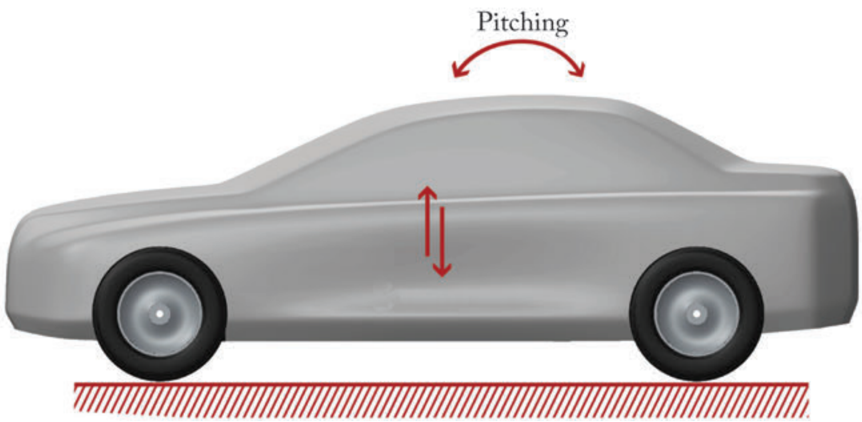


Figure 4.10: (b) Sprung mass movement—pitching.

A rolling tire can track along the ideal road profile, which is considered to be a simple sinusoidal shape shown in Figure 4.11. The road profile can be presented with the following basic formulation:

$$y(x) = d \sin\left(\frac{2\pi}{\lambda} x\right). \tag{4}$$

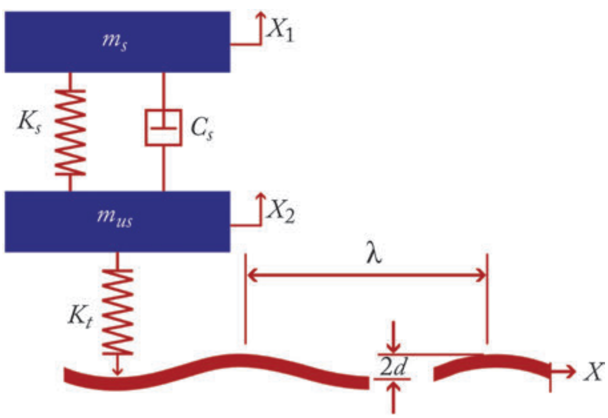


Figure 4.11: A tire on a simple sinusoidal road.

If we suppose the tire remains in contact with the road, we can further suppose:

$$x = ut. \tag{5}$$

By substituting Equation (5) in Equation (4):

$$y(t) = d \sin\left(\frac{2\pi u}{\lambda} t\right) = d \sin(\omega t)\ ,\ \omega = \frac{2\pi u}{\lambda}\quad . \tag{6}$$

Therefore, the road excitation frequency is related to the vehicle speed and it is conversely the road profile wavelength.

b. **Real Road Profile**

It is too unrealistic to think that real road profile is uniform and sinusoidal in shape. Any road profile can be decomposed into a series of sinusoidal functions using the Fourier transform. In Figure 4.12, we can see a typical signal on the left side that can be decomposed into four uniform sinusoidal waves. According to the previous section, dealing with a sinusoidal road profile is easy since we know how to precisely estimate his type of road induced vibration.

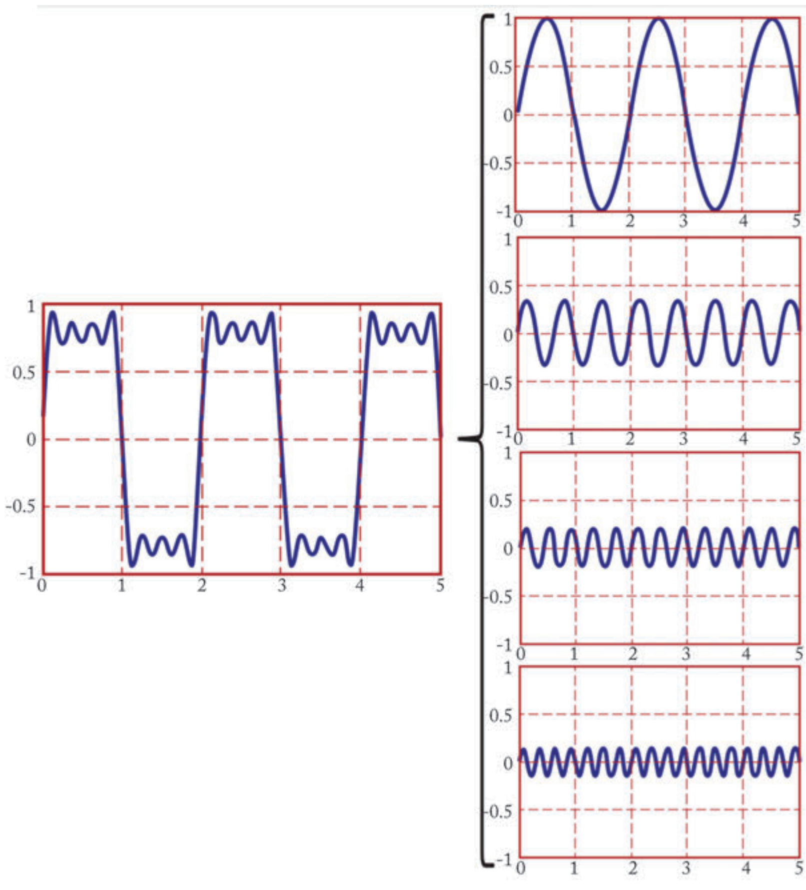


Figure 4.12: Decomposition of a typical signal to sine waves.

Given the Fourier transform, a real road's profile can be expressed based on a summation of an infinite number of sinusoidal functions as:

$$y(X) = d_0 + \sum_{k=1}^{\infty} d_k \sin(\omega_{pk} X + \varphi_k). \tag{7}$$

If we suppose that the tire remains in contact with the road, then Equation (7) can be rewritten as:

$$y(t) = d_0 + \sum_{k=1}^{\infty} d_k \sin(\omega_k t + \varphi_k), \tag{8}$$

where

$$\omega_k = \frac{2\pi u}{\lambda}.$$

c. **Standard Road Profile**

To assess the ride performance of a car, it is tested first on roads with standard profiles. These roads are discussed below.

Sinusoidal Road Profile: The most relevant kind of road profile that can be seen in test roads is the sinusoidal form. This type of road can be differed according to its spatial frequency. A typical road can have either a low spatial frequency with a long wavelength, or a high spatial frequency with a short wavelength. Figure 4.13 shows these two types of standard sinusoidal roads.

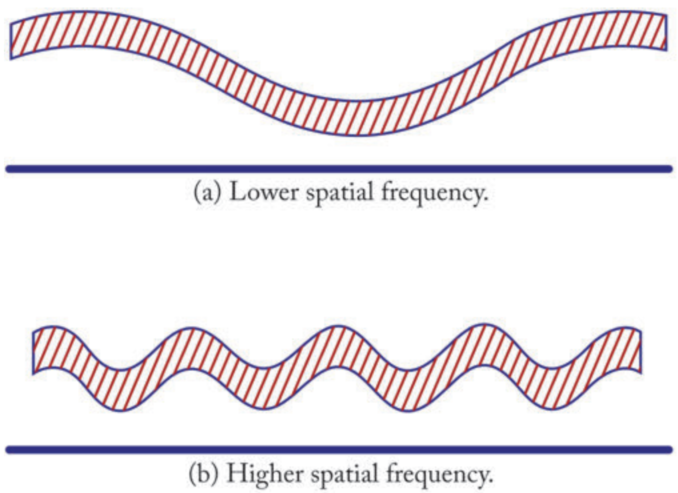


Figure 4.13: Two types of standard sinusoidal roads.

The two sides of the test road can be in phase with each other resulting in symmetric motion for each axle wheel. When the two sides of the road are anti-phase, the wheels on an axle move asymmetrically. The different phases between the left and right wheels on one axle are shown in Figure 4.14.

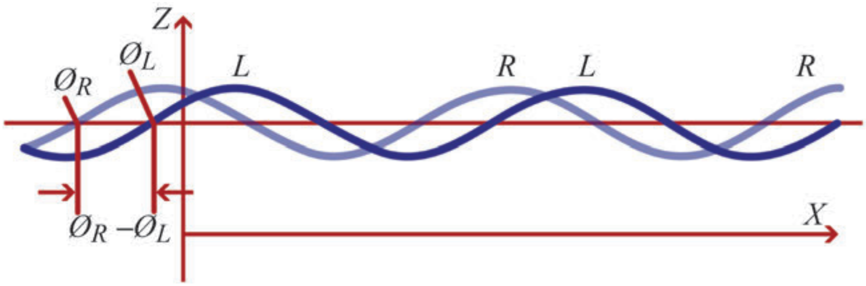


Figure 4.14: The difference between left and right wheels.

Thus, the road induced motion on the left and right wheels can be described by:

$$y_L = d_L \sin(\omega_{pL} X + \varphi_L) = d_L \sin(\omega_L t + \varphi_L), \text{ and} \quad (9)$$

$$y_R = d_R \sin(\omega_{pR} X + \varphi_R) = d_R \sin(\omega_R t + \varphi_R). \quad (10)$$

The standard sinusoidal road profiles have different wavelengths, and each wavelength is deliberately designed to comprehensively test a specific behavior of the suspension system. Table 4.1 summarizes different wavelengths and their major influences on a vehicle.

Table 4.1: Major influences on a vehicle with respect to the wide range of a sinusoidal road's wavelength

Characteristic	Wavelength	Influence on
Slopes	100 m < λ	Static
Undulations	1 m < λ <100 m	Dynamic ride
Roughness	10 mm < λ < 1 m	Dynamic NVH
Macro texture	1 mm < λ < 10 mm	Friction and noise
Micro texture	10 μm < λ < 1 mm	Friction
Material	Molecular	Friction

Isolated Bumps: Another type of standard road profile is isolated bumps, which unlike the sinusoidal road profile, is not continuous. The standard bumps can have different shapes and dimensions; some more important bumps have been named as rectangular, triangular, sine half-wave, and haversine. These standard bumps are shown in Figure 4.15.

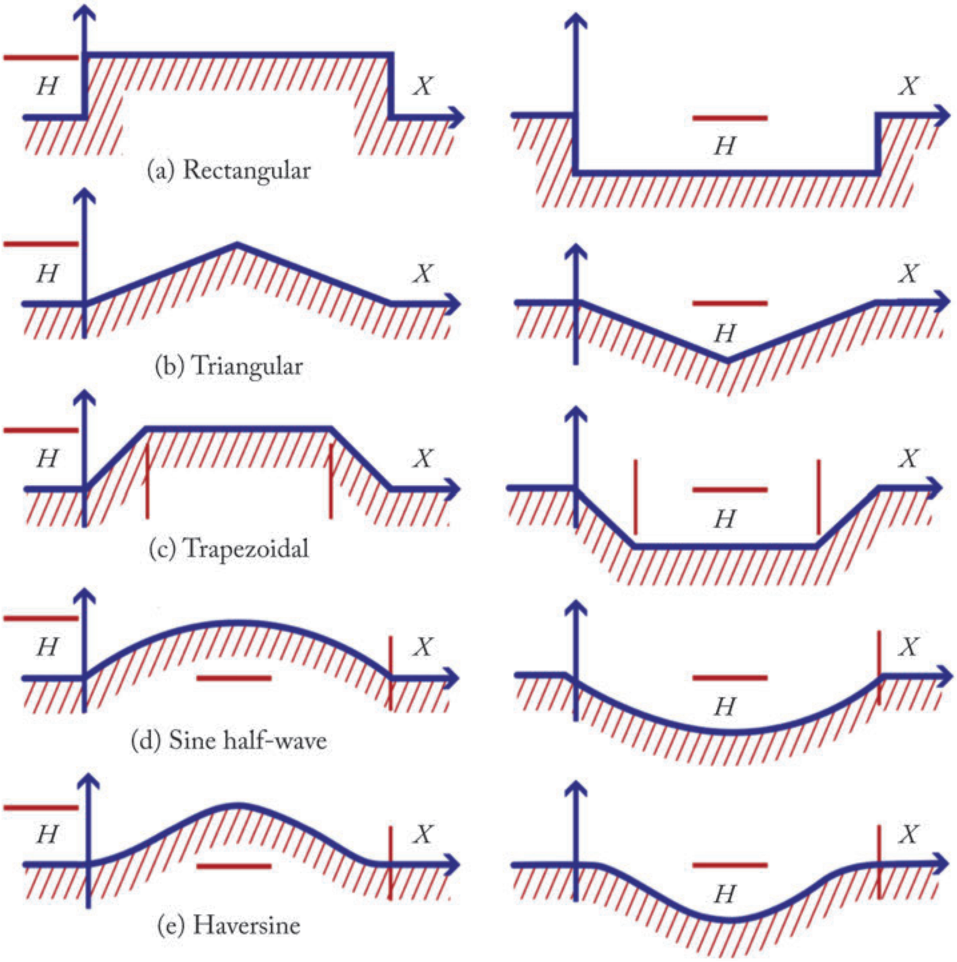


Figure 4.15: Different shapes of isolated bumps.

Road excitation according to bump shapes and dimensions can vary greatly, so the motion of the wheel also varies; a wheel's motion upward on a sine half-wave can be formulated as:

$$y = H \sin\left(\frac{\pi X}{L}\right);\ 0 \le X \le L. \tag{11}$$

While the equation of motion of a wheel on a haversine is defined as:

$$y = H\, hav\left(\frac{2\pi X}{L}\right);\ 0 \le X \le L, \tag{12}$$

where *hav* function is:

$$hav(\theta) = 1/2\ \{1 - \cos(\theta)\ \}. \quad (13)$$

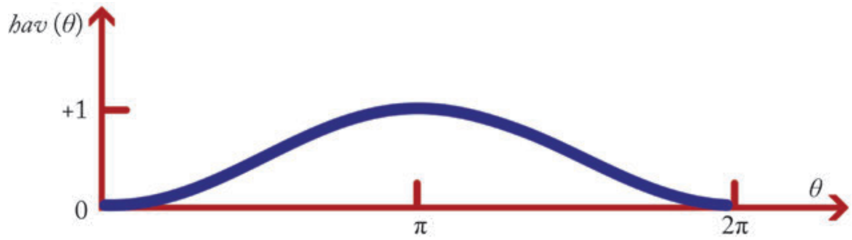


Figure 4.16: The *hav* function.

4.3 RIDE COMFORT EVALUATION

Defining a standard technique to evaluate the ride vibration of a vehicle is a debatable topic among automotive engineers. There is still no universal standard of evaluating ride vibrations. However, a number of standardized methods have been defined to assess the vibration exposure risks considering the human sensitivity to frequency and magnitude of vibration and exposure duration. The most common standard for evaluating ride comfort is ISO 2631-1. This standard evaluates human's exposure to whole-body vibration. As an example of vibration exposure assessment recommended by the ISO 2631-1, Figure 4.17 illustrates the decreased proficiency boundaries for vertical and horizontal vibrations in terms of root-mean-square (RMS) of the acceleration as a function of frequency for different exposure time. As it can be seen, the boundary lowers by increase in the daily exposure time.

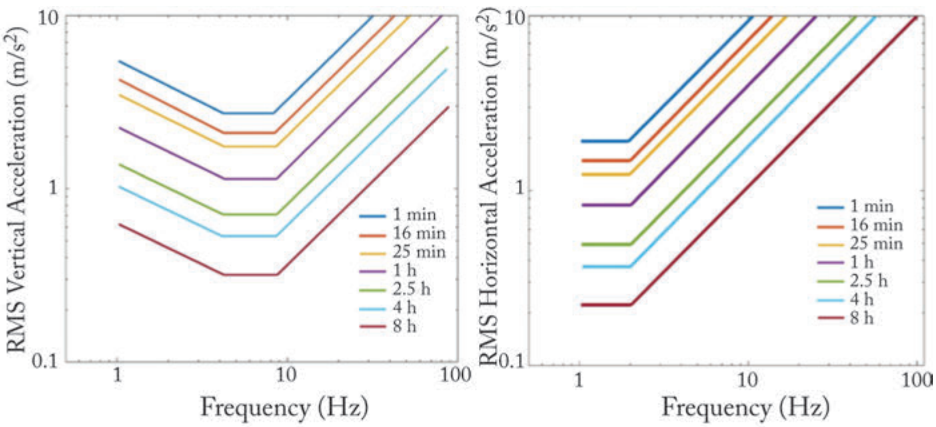


Figure 4.17: Whole-body vibration limits for decreased proficiency in vertical and horizontal directions, recommended by the ISO 2631.

The ISO-2631 also defines the frequency weightings for assessing of vertical and horizontal vibrations, while the exposure is expressed in terms of frequency-weighted root-mean-square (RMS) acceleration. As it is shown in Figure 4.18, the frequency-weighs used in this standardized exposure assessment method suggest that human body is most sensitive to horizontal vibration in the 0.5–2 Hz range, and to vertical vibration in the 4–10 Hz range. The weighted RMS acceleration can be formulated as:

$$RMS(\mathrm{a}_{w(\mathrm{x/y/z})}) = \sqrt{\frac{1}{T}\int_0^t [\mathrm{a}_{w(\mathrm{x/y/z})}]^2 dt}\,, \tag{14}$$

where $\mathrm{a}_{w(\mathrm{x/y/z})}$ denotes frequency-weighted acceleration along a given axis (x or y or z).

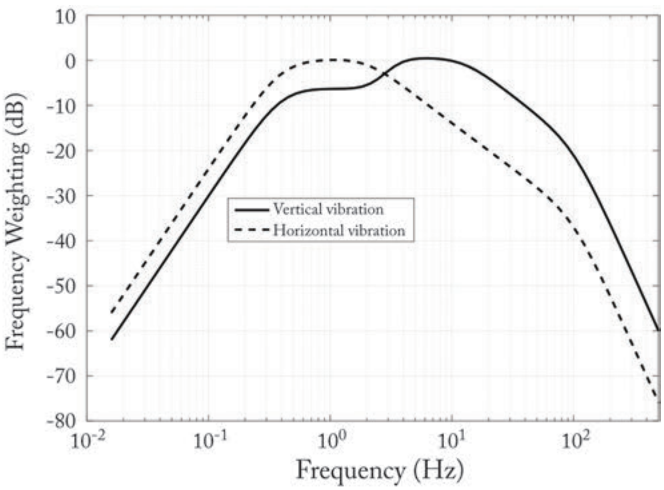


Figure 4.18: Frequency weighting curves for principal weighting, recommended by the ISO 2631.

4.4 VEHICLE VIBRATION MODEL

The spring and shock absorber are two of the main components in a suspension system and they play an important role in vehicle handling, and ride quality. Suspension designers usually use the natural frequency method to calculate the suitable spring stiffness. To implement the natural frequency method, there are four different ride models introduced below.

4.4.1 1 DOF QUARTER-CAR MODEL:

The simplest model for studying vehicle vibration is a 1 DOF quarter-car model. The sprung mass bounce motion is the only degree of freedom and the unsprung mass is eliminated. A solid mass indicating the sprung mass and a massless spring and damper are shown in Figure 4.19.

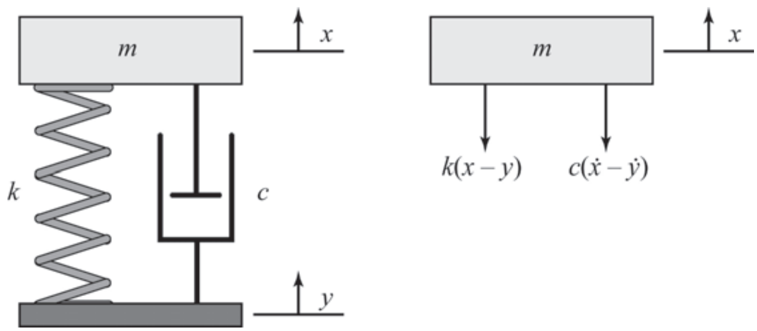


Figure 4.19: 1 DOF in a quarter car.

The model only considers a quarter car as a single wheel vehicle. Thus, the mass distribution on the front and rear axles are defined as:

$$m_f = \frac{b}{2L} M, \text{ and} \tag{15}$$

$$m_r = \frac{a}{2L} M, \tag{16}$$

where a, b, and L are the dimensions determined in Figure 4.20.

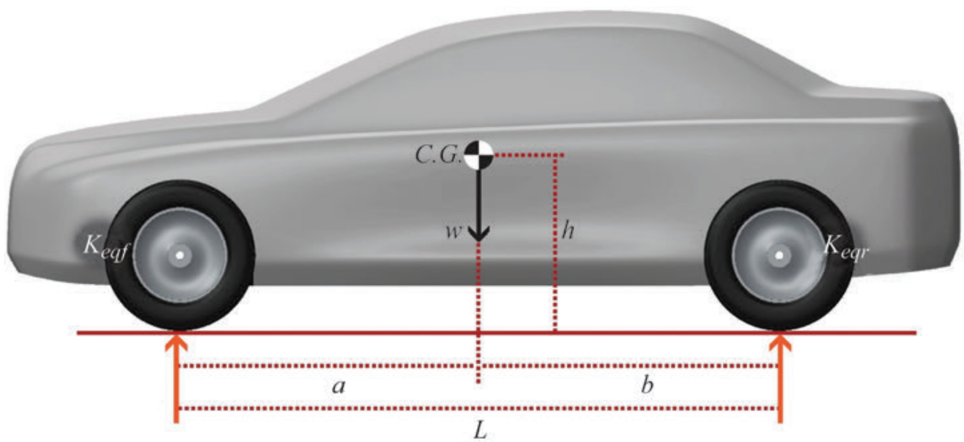


Figure 4.20: A vehicle's mass distribution over the front and rear axles.

Moreover, k is the equivalent stiffness of the tire vertical stiffness and suspension spring stiffness in series springs, which is defined as:

$$\frac{1}{k} = \frac{1}{k_s} + \frac{1}{k_t}. \tag{17}$$

According to Newton's second law:

$$\sum f = ma. \tag{18}$$

Only two forces act on the mass in a simple oscillatory system. One force is due to the spring and the other is due to the shock absorber. Therefore, by assuming x > y, the vibration equation of the mass becomes:

$$-c(\dot{x} - \dot{y}) - k(x - y) = m\ddot{x}, \tag{19}$$

or

$$m\ddot{x} + c(\dot{x} - \dot{y}) + k(x - y) = 0. \tag{20}$$

Transmissibility refers to the ratio of the maximum amplitude of the induced signal transmitted to the mass to the amplitude of the excitation signal. If transmissibility equals one, the mass will vibrate at the same amplitude as the excitation signal. If it is less than 1, the amplitude of the mass will be smaller than the excitation signal. In addition, if it is higher than 1, the system amplifies the mass vibration, and it will have a larger amplitude than the excitation signal. In order to calculate the transmissibility of a 1 DOF system, let us assume that the road profile is $y = Y \sin(\omega t)$; thus, Equation (20) will be rewritten as:

$$m\ddot{x} + c(\dot{x} - Y\omega \cos(\omega t)) + k(x - Y \sin \omega t) = 0, \tag{21}$$

or

$$m\ddot{x} + c\dot{x} + kx = c\omega Y \cos(\omega t) + kY \sin \omega t. \tag{22}$$

We can rewrite the above equation as:

$$\ddot{x} + 2\zeta\omega_n \dot{x} + \omega_n^2 x = 2\zeta\omega_n \omega Y \cos(\omega t) + \omega_n^2 Y \sin \omega t, \tag{23}$$

where the natural frequency and damping ratio are:

$$\omega_n = \sqrt{\frac{k}{m}} = 2\pi f_n, \tag{24}$$

$$\zeta = \frac{c}{2\sqrt{km}}. \tag{25}$$

If the sprung mass oscillates as:

$$x = X \sin(\omega t - \varphi_x), \tag{26}$$

then the steady state amplitude transmissibility can be obtained as:

$$\left|\frac{X}{Y}\right| = \sqrt{\frac{1 + (2\zeta r)^2}{(1 - r^2)^2 + (2\zeta r)^2}}\ , \tag{27}$$

where

$$r = \frac{\omega}{\omega_n}\,. \tag{28}$$

The response of the steady state amplitude transmissibility with respect to the proportion of the induced frequency and natural frequency is shown in Figure 4.21.

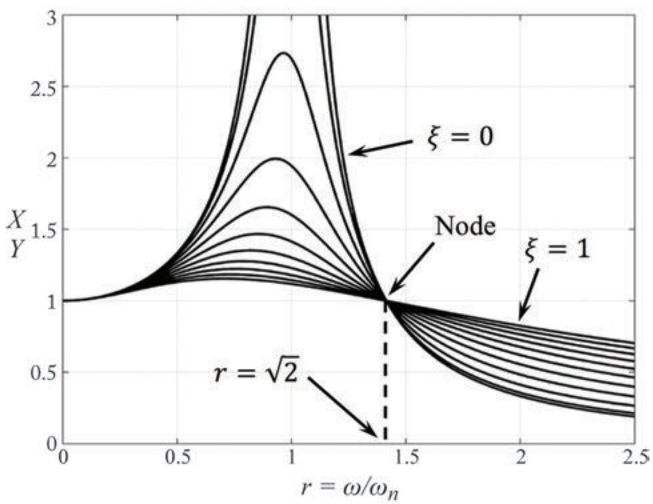


Figure 4.21: Displacement transmissibility of the sprung mass in Figure 4.19.

As shown in Figure 4.21, if the damping ratio, ζ, is zero and the frequency of excitation is equal to the natural frequency of the system ($\omega/\omega_n = 1$), the amplitude will increase to infinity. This is called resonance. Moreover, for any damping ratio, the transmissibility is 1 at frequency of $\sqrt{2}\,\omega_n$, which means that the mass amplitude is equal to the road profile amplitude. For frequencies higher than this, the transmissibility becomes less than 1.

In addition, in case we want to study the relative amplitude between the mass and road, we can consider:

$$z = Z \sin(\omega t - \varphi_z), \quad z = x - y. \tag{29}$$

Therefore, Equation (23) can be written as:

$$\ddot{z} + 2\zeta\omega_n \dot{z} + \omega_n^2 z = \omega^2 Y \sin(\omega t). \tag{30}$$

Finally, the steady state relative amplitude transmissibility can be obtained as:

$$\left|\frac{Z}{Y}\right| = \frac{r^2}{\sqrt{(1-r^2)^2 + (2\zeta r)^2}}. \tag{31}$$

The mass relative amplitude transmissibility with respect to the normalized excitation frequency (ω/ω_n) is plotted in Figure 4.22.

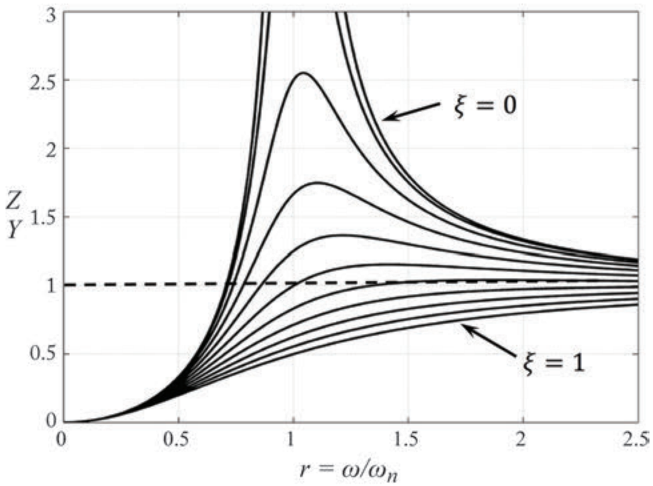


Figure 4.22: Relative displacement transmissibility of the sprung mass in Figure 4.19.

As shown in Figure 4.22, if the damping ratio, ζ, is zero, the relative amplitude will increase to infinity when the excitation frequency is equal to the system natural frequency ($\omega/\omega_n = 1$). As the excitation frequency increases, the transmissibility approaches 1, which means that the mass amplitude equals to the road profile amplitude. As seen in Figure 4.22, the transmissibility of systems with lower damping ratios is larger than systems with larger damping ratios.

If we want to study sprung mass acceleration, we can define the acceleration transmissibility as:

$$\left|\frac{\ddot{X}}{Y\omega_n^2}\right| = \frac{r^2\sqrt{1+(2\zeta r)}}{\sqrt{(1-r^2)^2 + (2\zeta r)^2}}. \tag{32}$$

The acceleration transmissibility with respect to the normalized excitation frequency (ω/ω_n) is shown in Figure 4.23.

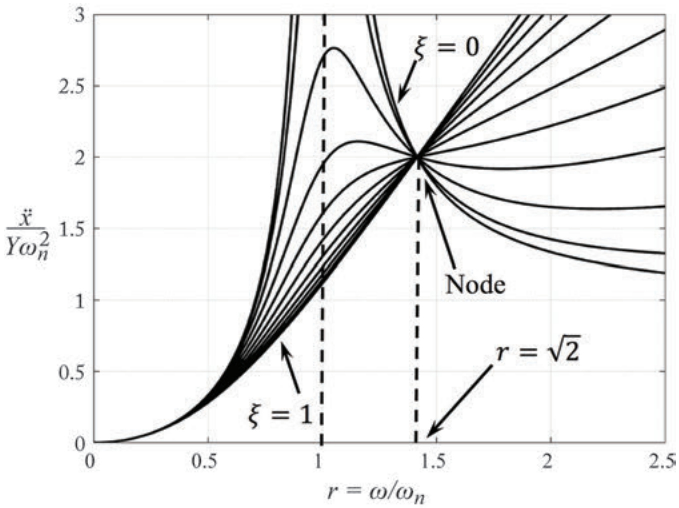


Figure 4.23: Acceleration transmissibility of the sprung mass in Figure 4.19.

According to Figure 4.23, the transmissibility increases to infinity when there is no damping in the system ($\zeta = 0$) at resonance frequency ($\omega/\omega_n = 1$). In addition, when $\omega/\omega_n = \sqrt{2}$, the transmissibility equals to 2 and independent from the damping ratio as shown in the figure. For excitation frequencies higher than $\omega = \sqrt{2}\,\omega_n$, the lower the damping ratio, the lower the acceleration of the mass. Regardless of the damping ratio, however, the acceleration transmissibility is larger than 1 for frequencies above $\sqrt{2}\,\omega_n$.

The phase and relative phase of the sprung mass can be defined as:

$$\varphi_x = arc\ \tan\left(\frac{2\zeta r^3}{1 - r^2 + (2\zeta r)^2}\right), \text{ and} \tag{33}$$

$$\varphi_z = arc\ \tan\left(\frac{2\ \zeta r}{1 - r^2}\right), \tag{34}$$

where φ_x and φ_z are the relative phase of sprung mass motion with respect to the excitation phase. The frequency response of the sprung mass phase is shown in Figure 4.24.

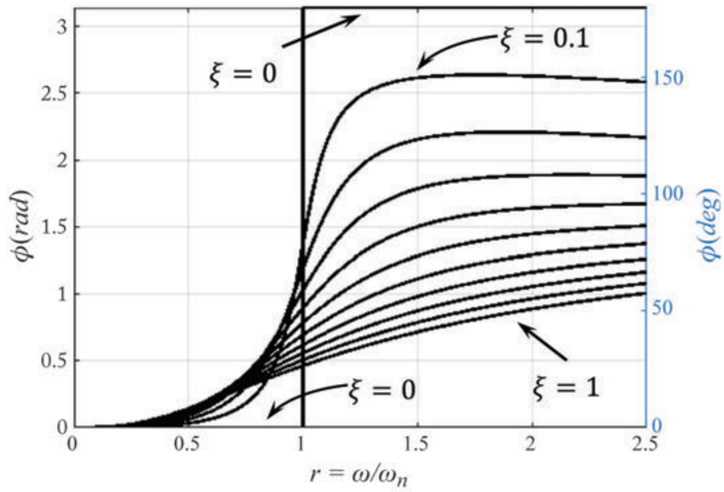


Figure 4.24: Frequency response of the sprung mass phase.

Example 2: Consider the quarter car model in Figure 4.19 with the following specifications.

a. Find the natural frequency and damping ratio of the suspension.

b. Plot the amplitude transmissibility, relative amplitude transmissibility, acceleration transmissibility, and the phase of the sprung mass.

c. Simulate and plot the time response (displacement and acceleration) of the 1 DOF quarter-car model to a typical haversine bump for a vehicle speed of 10 km/h. Discuss how the mass can affect the results.

Table 4.2: Specifications of a quarter car model

Symbol	Parameter	Value
m_s	Sprung mass	400 kg
m_u	Unsprung mass	45 kg
k_s	Spring stiffness	28 kN/m
c	Damping coefficient	2.5 KN/ms^{-1}
k_t	Tire stiffness	240 kN/m

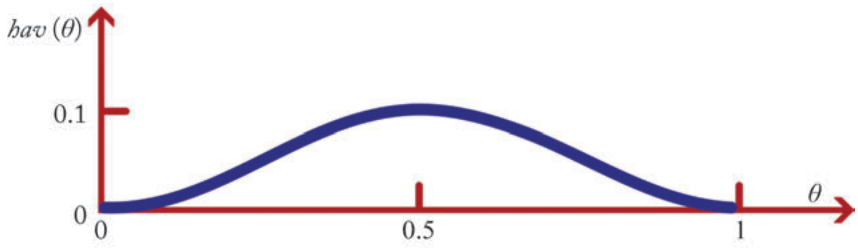


Figure 4.25: A typical Haversine.

Solution

a. To find the natural frequency and damping ratio of the suspension using a quarter car model or a 1 DOF model, the equivalent stiffness of the tire and spring should be calculated from Equation (17):

$$\frac{1}{k} = \frac{1}{28} + \frac{1}{240}$$

$$k = 25.075 \text{ kN/m}.$$

The natural frequency and damping ratio can be obtained using Equations (24) and (25), respectively:

$$\omega_n = \sqrt{\frac{k}{m^s + \mathrm{m}^\mathrm{u}}} = \sqrt{\frac{25075}{445}} = 7.51 \frac{\text{rad}}{s} = 1.19 \text{ Hz}$$

$$\zeta = \frac{2500}{2\sqrt{25075 \times 445}} = 0.3742.$$

b. The frequency response of the 1 DOF quarter-car model can be plotted using Equations (27), (31), (32), and (33).

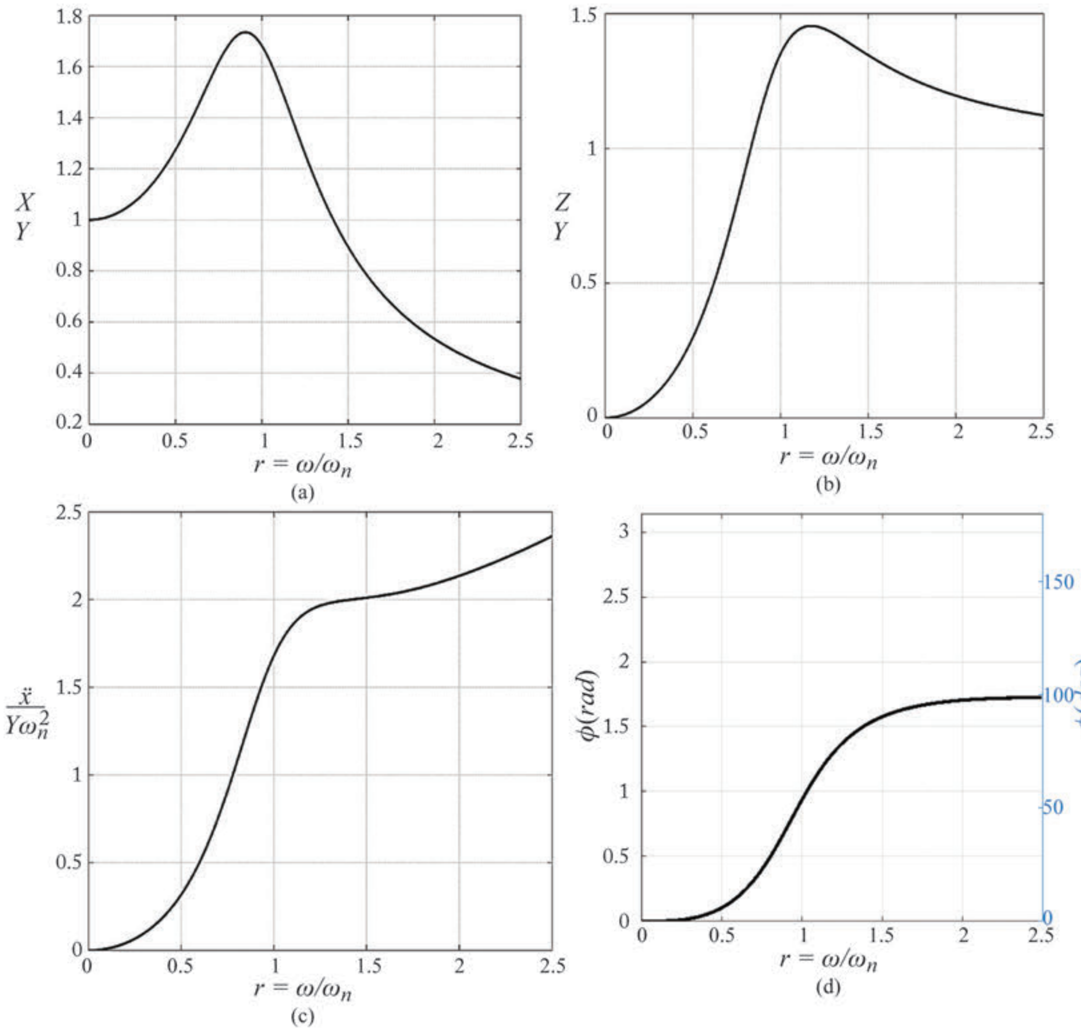


Figure 4.26: Frequency response of a 1 DOF quarter-car model: (a) amplitude transmissibility, (b) relative amplitude transmissibility, (c) acceleration transmissibility, and (d) sprung mass's phase.

c. The time response of the 1 DOF quarter model can be determined by solving the differential equation of this model presented in Equation (20). Figure 4.27 illustrates the time response of the model for three different values of mass. It can be seen that whenever the mass is lighter, the amplitude of vibration decreases more quickly because the lower masses require less kinematic energy so the shock absorber can turn this energy into heat more quickly.

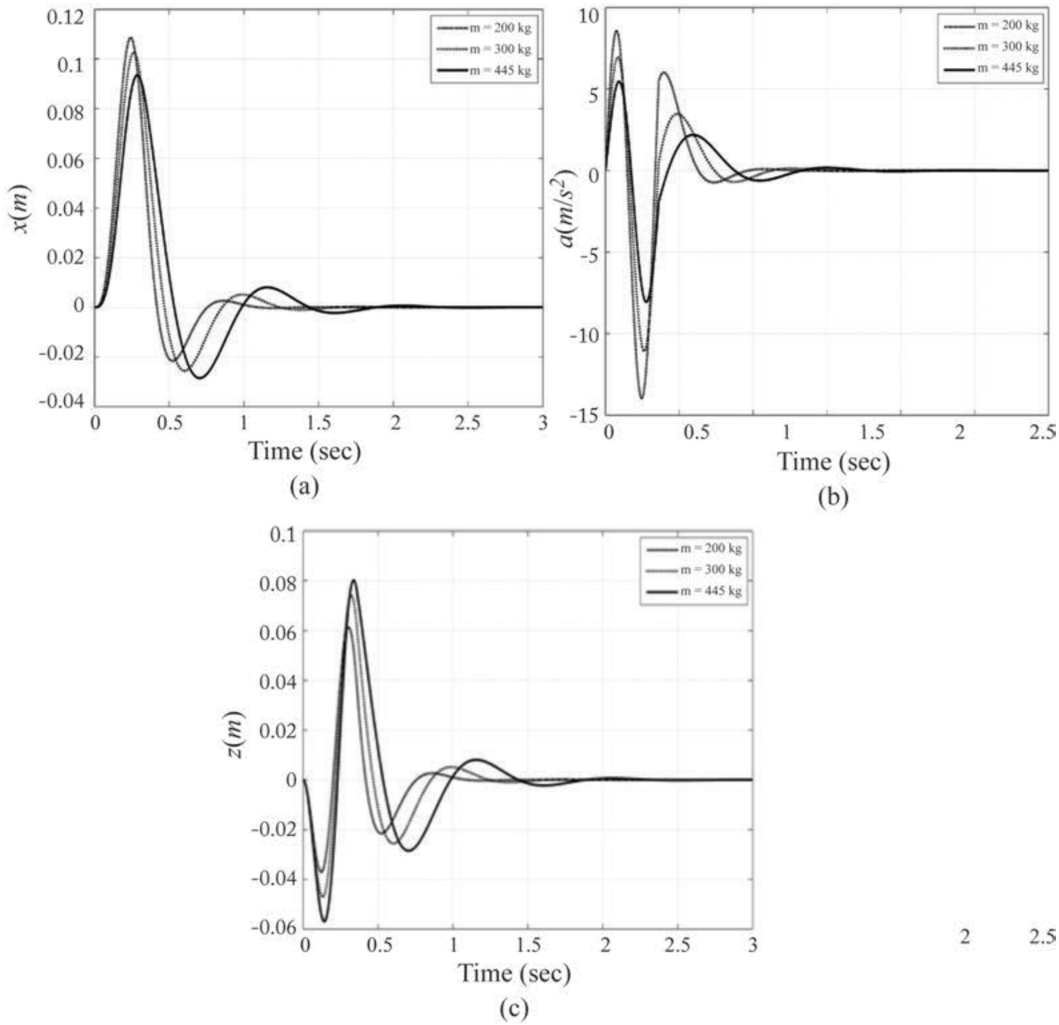


Figure 4.27: Time response of a 1 DOF quarter model to a typical haversine bump: (a) displacement, (b) acceleration, and (c) relative displacement.

Example 3: For the 1 DOF quarter model described in Example 1, consider two dampers with a higher damping coefficient of $\boldsymbol{c} = \mathbf{4010\ N/ms^{-1}}$ and a lower damping coefficient of $\boldsymbol{c} = \mathbf{1005\ N/ms^{-1}}$.

a. Find the frequency response of the model and compare the results with those obtained in Example 1.

b. After plotting the time response of the 1 DOF quarter-car model to a sine function with an amplitude of 0.1 m and frequencies of 0.5 Hz and 5 Hz, discuss how choosing a higher or lower damping coefficient can affect model response in low and high frequencies.

Solution

a. Using Equation (25) the damping ratio for ***c* = 4010 N/ms^{-1}** and ***c* = 1005 N/ms^{-1}** are 0.60 and 0.15, respectively.

The amplitude and acceleration transmissibility of the sprung mass for the above damping ratios and Example 2 are shown in Figure 4.28. As seen in the figure, increasing the damping ratio improves the ride quality at low frequencies. This means that both displacement transmissibility and acceleration transmissibility are lower at frequencies below $r = \sqrt{2}$. On the other hand, a suspension system with a lower damping ratio performs better at high frequencies (i.e., $r>\sqrt{2}$). As a result, choosing an optimum damping ratio for the suspension system is critical. This parameter should be determined such that the suspension system has an acceptable performance in both low and high frequencies.

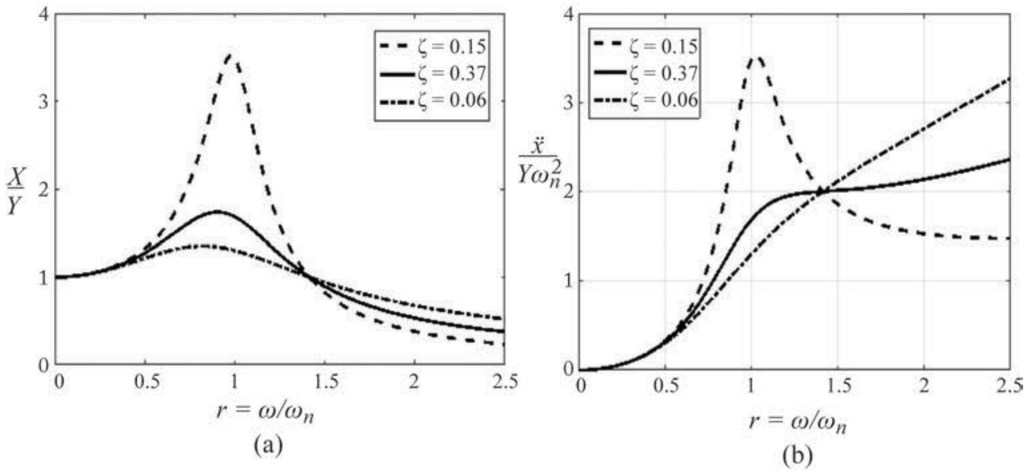


Figure 4.28: Frequency response comparison of a 1 DOF quarter-car model with different damping ratios: (a) amplitude transmissibility and (b) acceleration transmissibility.

b. As discussed in part (a), a higher damping ratio is more effective in low frequencies. The time response of the model to a sine wave with a low frequency of 0.5 Hz confirms that the suspension system with ζ=0.6 has the smallest displacement and acceleration. However, for a sine wave with a high frequency of 5 Hz, the lowest damping ratio (ζ=0.15) has the smallest displacement and acceleration. The time response of the suspension system with ζ=0.37 shows good performance in both cases. Therefore, ζ=0.37 is a better option for the damping ratio of this suspension system.

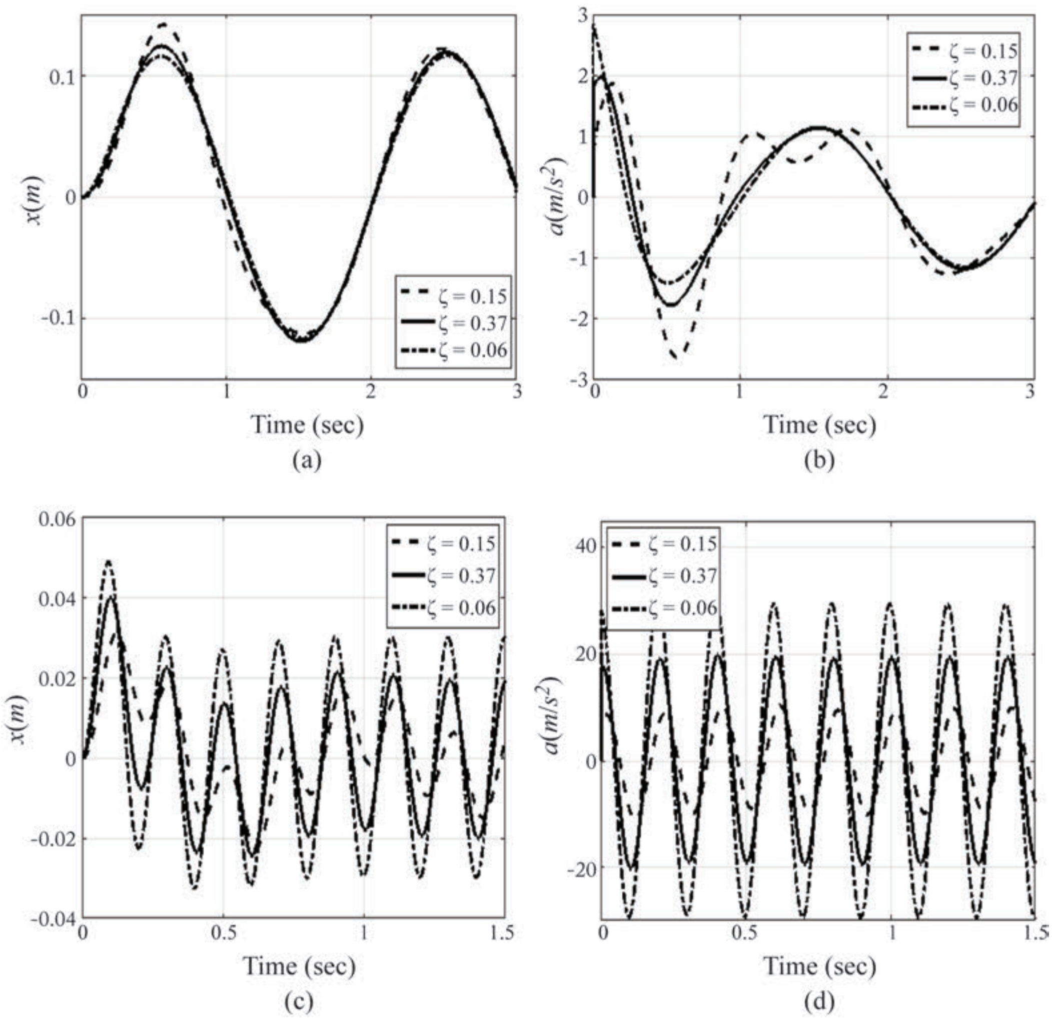


Figure 4.29: Time response comparison of a 1 DOF quarter-car model with different damping ratios: (a) displacement at low frequency, (b) acceleration at low frequency, (c) displacement at high frequency, and (d) acceleration at high frequency.

4.4.2 2 DOF QUARTER-CAR MODEL

A more complicated quarter-car model has 2 degrees of freedom. As shown in Figure 4.30, it includes both unsprung mass and sprung mass bounce motions. The unsprung mass m_{us} represents the wheel and its associated components, and the sprung mass m_s represents the corresponding vehicle's body mass to the wheel (approximately a quarter of the body mass). Sprung and unsprung bounce motions are described by x_1 and x_2, respectively. Suspension stiffness and damping coefficient are k_s and c_s. Tire vertical stiffness is k_t and its damping is usually negligible with respect to the suspension damping.

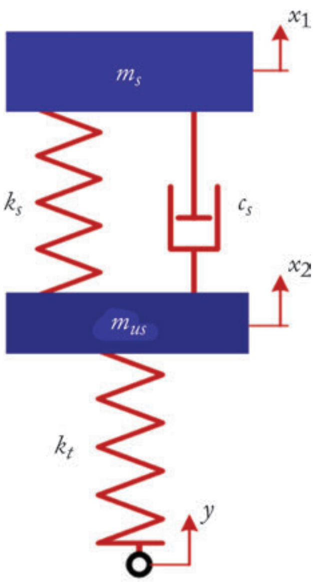


Figure 4.30: 2-DOF quarter-car ride model.

Using Figure 4.30 and assuming $x_2\,(t)>x_1\,(t)$, the equations of motion can be obtained using Newton's second law as:

$$m_s\,\ddot{x}_1\,(t) = c_s\,[\dot{x}_2\,(t) - x_1\,(t)] + k_s\,[x_2\,(t) - x_1(t)], \text{ and} \tag{35}$$

$$m_{us}\,\ddot{x}_2\,(t) = -c_s\,[\dot{x}_2\,(t) - x_1\,(t)] - k_s\,[x_2\,(t) - x_1(t)] - k_t\,[x_2\,(t) - y(t)]. \tag{36}$$

To find the frequency response of the system, we assume:

$$x_1\,(t) = X_s \sin(\omega t - \varphi_1), \tag{37}$$

$$x_2\,(t) = X_{us} \sin\,(\omega t - \varphi_2\,), \text{ and}$$

$$y\,(t) = Y \sin\,\omega t. \tag{38}$$

By replacing Equations (37) and (38) in Equations (35) and (36), we obtain:

$$(-m_s\,\omega^2 + c_s\omega + k_s)X_s + (-c_s\omega - k_s)\,X_{us} = 0, \text{ and} \tag{39}$$

$$(-c_s\omega - k_s)\,X_s + (-m_{us}{}^2 + c_s\omega + k_s + k_t)\,X_{us} = k_t Y\,. \tag{40}$$

To determine the undamped natural frequency of the system, it is assumed that $c_s = 0$ and $Y = 0$. To have a non-trivial solution for the system of Equations (39) and (40), we need to have:

$$det \begin{bmatrix} -m_s\,\omega^2 + k_s & -\,k_s \\ -\,k_s & m_{us}\,\omega^2 + k_s + k_t \end{bmatrix} = 0. \tag{41}$$

This leads to:

$$m_s\, m_{us}\, \omega^4 - [(k_s + k_t\,)m_s + k_s\, m_{us}]\omega^2 + k_s\, k_t \;=0. \tag{42}$$

Thus, the natural frequency can be determined as:

$$\omega^2 = \frac{k_s + k_t}{2m_{us}} + \frac{k_s}{2m_s} \pm \frac{\sqrt{(k_s + k_t)^2\, m_s^2 + m_{us}^2\;\; k_s^2\; -2(k_t - k_s)\, k_s\, m_s\, m_{us}}}{2m_s\, 2m_{us}}\,. \tag{43}$$

Since k_t is vastly bigger than k_s, it is assumed that:

$$k_s + k_t \approx k_t - k_s \approx k_t\,. \tag{44}$$

By considering Equations (43) and (44), the approximate natural frequencies of the system become:

$$\omega_{n_s} = \sqrt{\frac{k_s}{m_s}}\text{, and} \tag{45}$$

$$\omega_{n_{us}} = \sqrt{\frac{k_s}{m_s}}\,. \tag{46}$$

Usually, for a modern passenger car, the natural frequency of the sprung mass is approximately 1 Hz, and the natural frequency of the unsprung mass is approximately 10 Hz.

Assuming m_{sv} as the vehicle sprung mass, the front and rear axle mass can be determined by:

$$m_{s_f} = \frac{m_{s_v}\, b}{L}\text{, and} \tag{47}$$

$$m_{s_r} = \frac{m_{s_v}\, a}{L}\,, \tag{48}$$

where m_{s_f} and m_{s_r} are the mass distribution over the front and rear axle. By using Equations (45), (47), and (48), the spring stiffness in the front and rear axles will become:

$$k_{s_{front}} = \frac{m_{sv}\, b}{2L}\,(\omega_{ns}\,)^2\text{, and} \tag{49}$$

$$k_{s_{rear}} = \frac{m_{s_v}\, a}{2L}\,(\omega_{n_s}\,)^2. \tag{50}$$

To improve the vehicle pitch motion, the sprung mass natural frequency in the rear axle is bigger than in the front axle, therefore, the natural frequency of the front axle is chosen to be 1 Hz and the natural frequency of the rear axle is selected to be 1.2 Hz.

In order to determine amplitude transmissibility, Equations (39) and (40) are used and the sprung mass amplitude transmissibility becomes:

$$\mu = \left|\frac{X_s}{Y}\right|, \qquad \mu^2 = \frac{4\xi^2 r^2 + 1}{Z_1^2 + Z_2^2}, \tag{52}$$

where

$$Z_1 = [r^2 (r^2 \alpha^2 - 1) + (1 - (1 + \varepsilon) r^2 \alpha^2)], \tag{53}$$

$$Z_2 = 2\zeta r(1 - (1 + \varepsilon) r^2 \alpha^2), \tag{54}$$

$$\xi = c_s / (2m_s \omega_s)) , \tag{55}$$

$$\alpha = {}^{\omega_s}/\omega_u , \tag{56}$$

$$\varepsilon = {}^{m_s}/m_u , \text{ and} \tag{57}$$

$$r = \frac{\omega}{\omega_n} . \tag{58}$$

The response of the sprung mass amplitude transmissibility with respect to the proportion of the induced frequency and the natural frequency is shown in Figure 4.31.

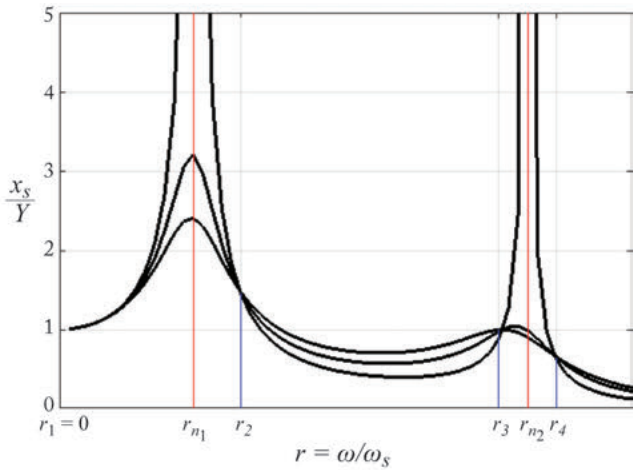


Figure 4.31: The sprung mass amplitude transmissibility.

The system has 2 DOF and as Figure 4.31 shows, it has two natural frequencies where the amplitude of the system without a damping ratio will go to infinity. The natural frequencies, r_{n_1} and r_{n_2}, can be determined as:

$$r_{n_1} = \sqrt{\frac{1}{2\alpha^2}(1 + (1 + \varepsilon)\alpha^2 - \sqrt{(1 + (1 + \varepsilon)\, \alpha^2)^2 - 4\alpha^2)}} \text{ , and} \tag{59}$$

$$r_{n_2} = \sqrt{\frac{1}{2\alpha^2}(1 + (1 + \varepsilon)\,\alpha^2 + \sqrt{(1 + (1 + \varepsilon)\,\alpha^2\,)^2 - 4\alpha^2)}}\,. \tag{60}$$

As it has been shown in Figure 4.31, in r_2, r_3, and r_4, the transmissibility is independent of the damping values. Furthermore, between r_2 and r_3, a sprung mass with lower damping ratio has smaller transmissibility. However, it has a bigger transmissibility in the range of r_1 and r_2.

The unsprung mass amplitude transmissibility is:

$$\tau = \left|\frac{X_\mathrm{u}}{Y}\right|, \qquad \tau^2 = \frac{4\xi^2\,r^2 + 1 + r^2\,(r^2 - 2)}{Z_1^2 + Z_2^2}\,. \tag{61}$$

In addition, the relative amplitude transmissibility between the sprung and unsprung masses can be defined as:

$$\eta = \left|\frac{Z}{Y}\right|, \qquad \eta^2 = \frac{r^4}{Z_1^2 + Z_2^2}\,. \tag{62}$$

Moreover, the acceleration transmissibility for the sprung and unsprung masses can be defined as:

$$\left|\frac{\ddot{X}_s}{Y\omega_s^2}\right| = r^2\,\alpha^2\mu, \text{ and} \tag{63}$$

$$\left|\frac{\ddot{X}_u}{Y\omega_u^2}\right| = r^2\,\alpha^2\tau. \tag{64}$$

The response of the sprung mass acceleration transmissibility with respect to the proportion of the induced frequency and natural frequency is shown in Figure 4.32. The system has 2 DOF, so it has two natural frequencies and the two jumps are shown in Figure 4.32, which also indicates how systems with lower damping ratios have more critical situations in these frequencies even though they have smaller transmissibility in other frequencies. One of the natural frequencies is approximately 1, which refers to the sprung mass, and the other is approximately 10, which is associated with the unsprung mass.

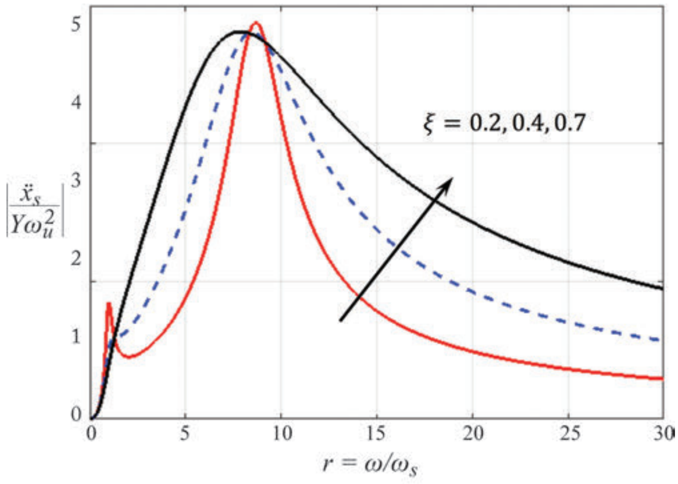


Figure 4.32: The response of the steady state's sprung acceleration transmissibility.

Example 4: Using the suspension specifications provided in Example 2 and a 2 DOF quarter-car model:

a. Find the natural frequencies and the damping ratio of the suspension.

b. Plot the frequency response (amplitude transmissibility and acceleration transmissibility) of the sprung mass and unsprung mass along with the relative amplitude transmissibility.

c. Simulate and plot the time response (sprung/unsprung mass displacement and acceleration as well as relative displacement) of a 2 DOF quarter-car model to a typical haversine (Figure 4.25) bump for vehicle speeds of 10 km/h.

Solution

a. Natural frequencies in a 2 DOF quarter-car model can be obtained using Equations (45) and (46):

$$\omega_s = \sqrt{\frac{28000}{400}} = 8.36\,\frac{rad}{\mathrm{s}} = 1.33 \text{ Hz, and}$$

$$\omega_u = \sqrt{\frac{240000}{45}} = 73.03\,\frac{rad}{s} = 11.62 \text{ Hz.}$$

Using Equations (56)–(60), the natural frequencies r_{n_1} and r_{n_2} can be determined as:

$$\alpha = \frac{\omega_s}{\omega_u} = \frac{8.36}{73.03} = 0.11,$$

$$\varepsilon = {}^{m_s}/m_u = 8.88,$$

$$r_{n1} = \sqrt{\frac{1}{2(0.11)^2}(1 + (1 + 8.88)\,(0.11)^2) - \sqrt{(1 + (1 + 8.88)\,(0.11)^2))^2 - 4(0.11)^2}} = 0.95 \text{ Hz},$$

$$r_{n1} = \sqrt{\frac{1}{2(0.11)^2}(1 + (1 + 8.88)\,(0.11)^2) + \sqrt{(1 + (1 + 8.88)\,(0.11)^2))^2 - 4(0.11)^2}} = 9.23 \text{ Hz}.$$

The damping ratio in this model can be calculated by Equation (55):

$$\xi = c_s/(2m_s\omega_s) = \frac{2500}{2 \times 400 \times \sqrt{\frac{28000}{400}}} = 0.3735.$$

b. As shown in Figure 4.33, the frequency response of the 2 DOF quarter-car model can be plotted using Equations (52), (61)–(64). As expected, the highest amplitude of the sprung mass occurs at the first natural frequency, and the highest amplitude of the unsprung mass happens at the second natural frequency.

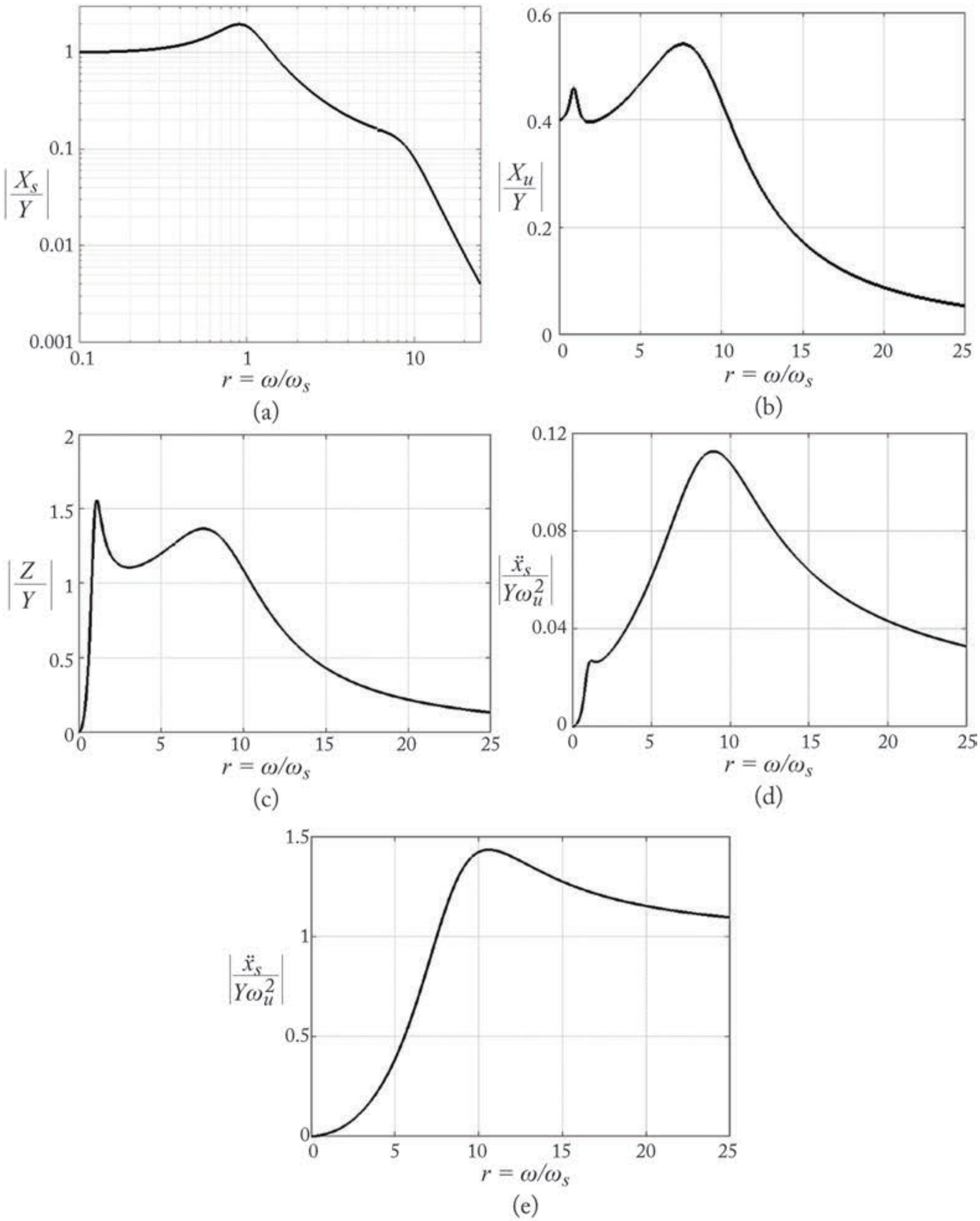


Figure 4.33: Frequency response of a 2DOF quarter-car model: (a) sprung mass amplitude transmissibility, (b) unsprung mass amplitude transmissibility, (c) relative amplitude transmissibility, (d) sprung mass acceleration transmissibility, and (e) unsprung mass acceleration transmissibility.

c. The time response of the 2 DOF quarter-car model can be determined by solving the differential equations of this model. As it is shown in Figure 4.34, the haversine isolated bump induces the sprung and unsprung masses to vibrate, and after some oscillation, the shock absorber dampens the kinematic energy.

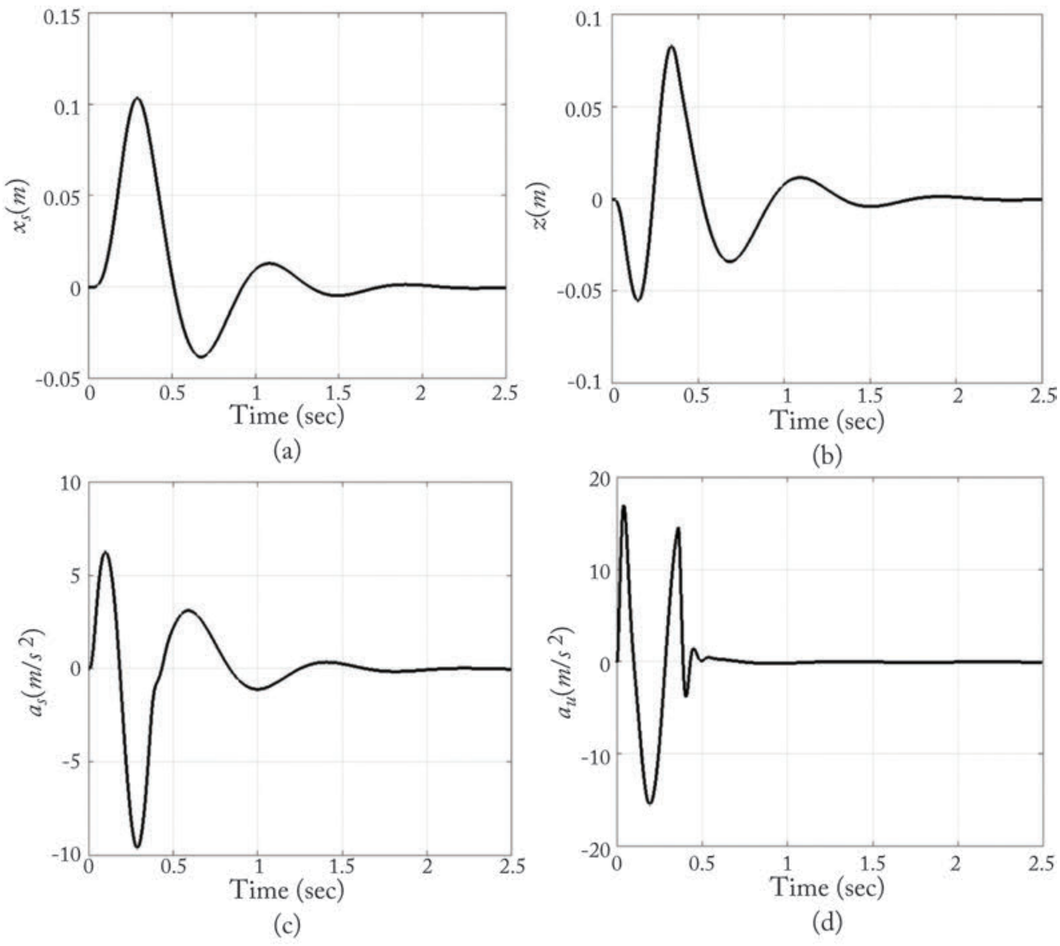


Figure 4.34: Time response of a 2 DOF quarter-car model to a typical haversine bump: (a) displacement of sprung mass, (b) relative displacement, (c) acceleration of sprung mass, and (d) acceleration of unsprung mass.

4.4.3 HALF-CAR MODEL

A half car model includes the bounce and pitch motions of the body, as shown in Figure 4.35. To determine the undamped natural frequencies, the suspension and tire damping are neglected and the equivalent suspension and tire stiffness, k_{eq} is defined as:

$$k_{eq} = \frac{k_s\, k_t}{k_s + k_t} \,. \tag{65}$$

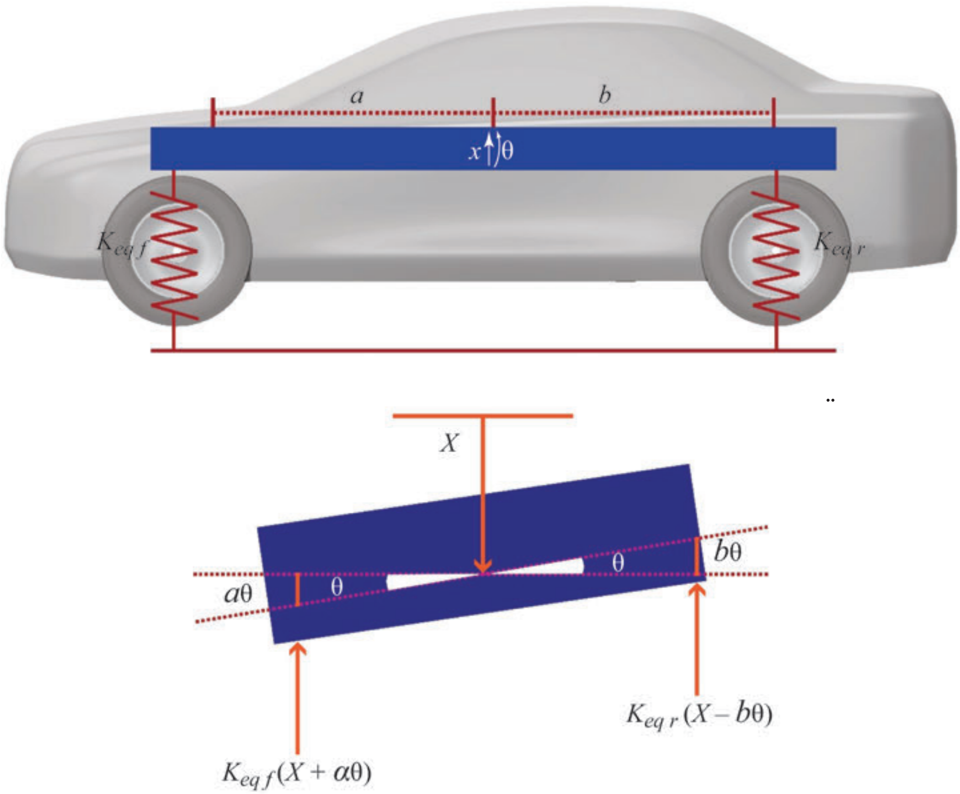


Figure 4.35: Half-car ride model.

The acting forces on the body of the vehicle are shown in Figure 4.35. According to Newton's second law, the equations of motion are written as:

$$\sum F_x = ma_x \text{ , and} \tag{66}$$

$$\sum M_{C.G} = I_{yy}\,\ddot{\theta} \quad , \tag{67}$$

where m and I_{yy} are half of the vehicle's mass and pitch mass inertia, respectively. Equations (66) and (67) can be rewritten as:

$$-k_{eq\,f}(x + a\theta) - k_{eq\,r}(x - b\theta) = m\ddot{x}\text{ , and} \tag{68}$$

$$-k_{eq\,f}(x + a\theta)a + k_{eq\,r}(x - b\theta)b = I_{C.G}\,\ddot{\theta}\text{ .} \tag{69}$$

By simplification:

$$m\ddot{x} + (k_{eq\,f} + k_{eq\,r})x + (k_{eq\,f}\,a - k_{eq\,r}\,b)\theta = 0\text{ , and} \tag{70}$$

$$I_{C.G}\,\ddot{\theta} + (k_{eq\,f}a^2 + k_{eq\,r}\,b^2)\theta + (k_{eq\,f}\,a - k_{eq\,r}\,b)x = 0\text{ .} \tag{71}$$

Using the moment equilibrium around the CG of the vehicle in steady states:

$$k_{eq\,f}\,a - k_{eq\,r}\,b = 0, \tag{72}$$

and considering Equations (70), (71), and (72), we obtain:

$$\omega_{n,T} = \sqrt{\frac{k_{eq\,f} + k_{eq\,r}}{m}}\text{ ,} \tag{73}$$

$$\omega_{n,R} = \sqrt{\frac{k_{eq\,f}\;a^2 + k_{eq\,r}\,b^2}{I_{C.G}}}\text{ ,} \tag{74}$$

$$k_{eq\,f} = \frac{I_{C.G}\,\omega_{n,R}{}^2 - b^2\,m\omega_{n,T}{}^2}{a^2 - b^2}\text{ , and} \tag{75}$$

$$k_{eq\,r} = m\omega_{n,T}{}^2 - \frac{I_{C.G}\,\omega_{n,R}{}^2 - b^2\,m\omega_{n,T}{}^2}{a^2 - b^2}\text{ .} \tag{76}$$

Usually, the bounce natural frequency $\omega_{\mathrm{n,T}}$ and pitch natural frequency $\omega_{\mathrm{n,R}}$ have a suggested value of approximately 1 Hz.

Example 5: Consider an electric vehicle with two in-wheel motors. The car's batteries are accommodated in the vehicle floor structure. The vehicle mass is 920 kg and its pitch inertia I_{yy} is 745 kg.m^2, while each wheel and its associated mass is 12 kg, and each electric motor mass is 20 kg. The vehicle wheelbase is 1.8 m and the distance between the center of gravity and rear axle is b = 0.895 m. The tir vertical stiffness is k_t = 220 kN/m.

a. Find the front and rear spring stiffness using a quarter-car model.

b. Find the front and rear spring stiffness using a half-car model.

Solution

a. The vehicle wheel base is 1.8 m and b = 0.895 m, so a = 0.905 m.

Electric motors are installed in the rear wheel and are counted in the unsprung mass, so in the quarter-car model:

m_{us_f} = 12 kg, $\quad m_{us_r}$ = 32 kg, and

$$m_{s_v} = m - m_{us} = 920 - 2\times12 - 2\times32 = 832 \text{ kg}.$$

Usually, the spring stiffness in the rear axle is bigger than in the front axle. So, the natural frequency in the front axle is assumed to be 1 Hz, and in the rear axle, it is selected as 1.2 Hz.

Note: 1 Hz = 2π rad/sec

By using Equations (49) and (50):

$$k_{sfront} = \frac{m_{sv}\, b}{2L}(\omega_{n_s})^2 = \frac{832 \times 0.895}{2 \times 1.8}(2\pi)^2 \approx 8{,}166 \text{ N/m, and}$$

$$k_{rear} = \frac{m_{sv}\, a}{2L}(\omega_{n_s})^2 = \frac{832 \times 0.905}{2 \times 1.8}(1.2 \times 2\pi)^2 \approx 11{,}890 \text{ N/m}.$$

b. The bounce and pitch natural frequencies are selected as 1 Hz. So, by using Equations (75) and (76):

$$k_{eq\,f} = \frac{I_{CG}\omega_{n,R}{}^2 - b^2\, m\omega_{n,T}{}^2}{a^2 - b^2} = \frac{745.18 \times (2\pi)^2 - 0.895^2 \times 920 \times (2\pi)^2}{2(0.905^2 - 0.895^2)} \approx 9033 \text{ N/m},$$

$$k_{eq\,r} = m\omega_{n,T}{}^2 - \frac{I_{CG}\omega_{n,R}{}^2 - b^2\, m\omega_{n,T}{}^2}{2(a^2 - b^2)} = m\omega_{n,T}{}^2 - k_{eq\,f} = 920\times(2\pi)^2 - k_{eq\,f} \approx 9127$$

N/m.

According to Equation (65):

$$k_{s_f} = \frac{k_t k_{eq\,f}}{kt - k_{eq\,f}} = \frac{220000 \times 9033}{220000 - 9033} \approx 9420 \text{ N/m},$$

$$k_{s_r} = \frac{k_t k_{eq\,r}}{kt - k_{eq\,r}} = \frac{220000 \times 9127}{220000 - 9127} \approx 9522 \text{ N/m}.$$

4.4.4 FULL VEHICLE MODEL

A full vehicle ride model has 7 degrees of freedom including bounce, pitch, and roll motions for the vehicle body, and 4 bounce motions for the wheels. This model is too complicated for natural frequency analysis but it is applicable for accurate computer simulations. Figure 4.36 shows a full car model.

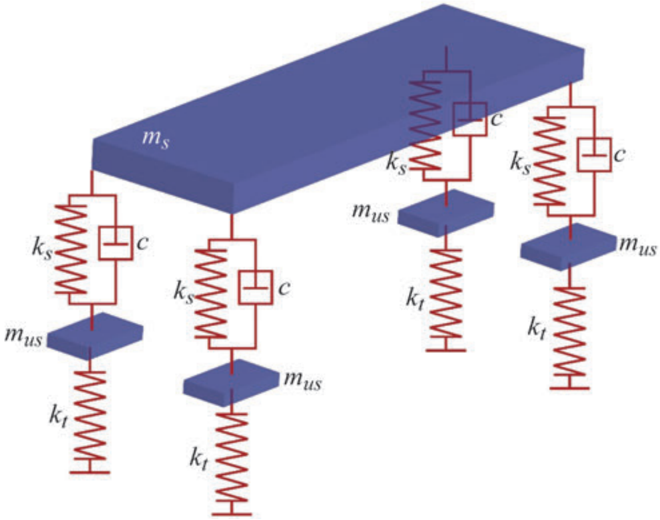


Figure 4.36: Full vehicle ride model.

4.5 INSTALLATION EFFECTS

In the above models, the spring is supposed to be installed directly above the tire. However, in reality, the spring is connected to the control arm that needs to be considered in the calculation of the effective spring and damping coefficient. Using Figure 4.37, we can write:

$$\frac{\Delta_z}{\Delta_{za}} = \frac{l}{l_1} \; ; \tag{77}$$

then

$$\Delta z_a = \frac{l_1}{1}\,\Delta z. \tag{78}$$

The potential energy that is stored in the theoretical and real springs should equal:

$$E = E_a \text{, and} \tag{79}$$

$$\frac{1}{2} k_s \,\Delta z^2 = \frac{1}{2} k_{sa}\,\Delta z_a{}^2. \tag{80}$$

By substituting Equation (78) in Equation (80):

$$k_{sa} = k_s \left(\frac{l}{l_1}\right)^2. \tag{81}$$

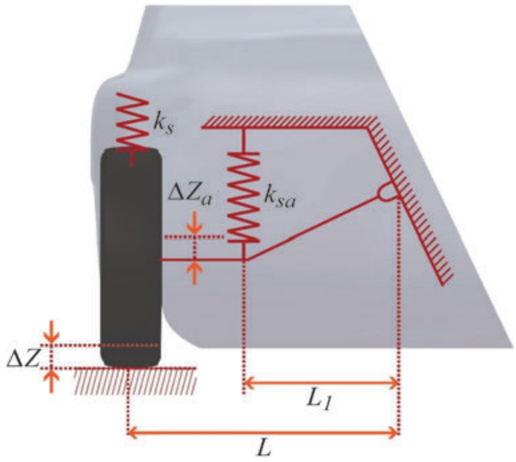
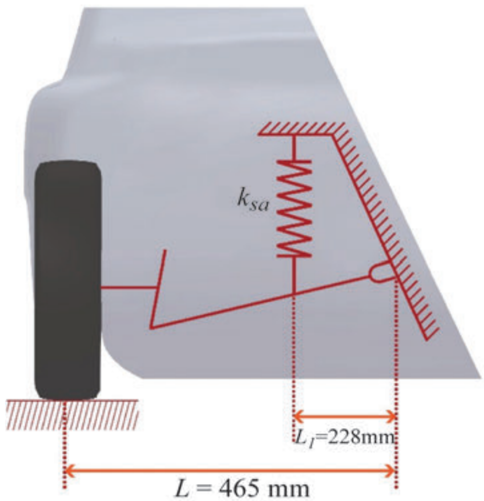


Figure 4.37: Spring installation effects.

Example 6: For the front suspension of the vehicle in Example 5(a), find the actual spring stiffness based on the following figure:



Solution: According to Example 5(a), the calculated front spring stiffness is 8,166 N/m. By using Equation (81), the actual spring stiffness is:

$$k_{sa} = k_s \left(\frac{l}{l_1}\right)^2 = 8166\left(\frac{0.465}{0.288}\right)^2 \approx 21287 \text{ N/m}.$$

4.6 ANTI-ROLL BAR SIZING

The main function of the anti-roll bar is to control the vehicle's roll during cornering maneuvers. To study the vehicle's roll motion, a 1 DOF vehicle roll-model is shown in Figure 4.38. The body rolls about the roll center RC while the lateral inertia force acts on the CG during cornering. The roll-model can be represented as an inverted pendulum. The spring and the shock absorber are modeled as a torsional spring kta and a torsional damper cta, where they, respectively, represent torsional stiffness and torsional damping of the vehicle.

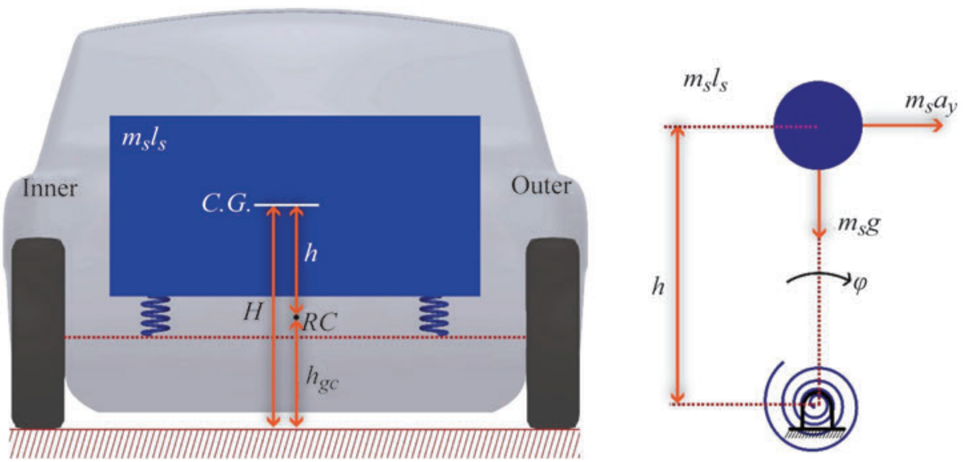


Figure 4.38: Vehicle roll model.

By using the D'Alembert principle, the momentum equation about the CG is:

$$-m_s a_y h \cos\varphi - m_s gh \sin\varphi + k_t\,\varphi + c_t\,\varphi = -I_{zs}\,\ddot{\varphi}\,, \quad (82)$$

where I_{zs} is the roll momentum of inertia of the sprung mass, and it is defined as:

$$I_{zs} = I + mh^2, \quad (83)$$

where I is the roll momentum of inertia of the sprung mass around the CG. For a small roll angle, φ:

$$\sin\varphi \approx \varphi,\ \ \cos\varphi \approx 1. \quad (84)$$

As such, Equation (82) is rewritten as:

$$I_{zs}\,\ddot{\varphi} + c_t\,\dot{\varphi} + (k_t - m_s gh)\varphi = m_s\,a_y\,h. \quad (85)$$

The roll angle φ is determined numerically by solving Equation (85). In the steady state condition, we can suppose:

$$\ddot{\varphi} = \dot{\varphi} = 0. \quad (86)$$

As such, Equation (85) is simplified to:

$$(k_t - m_s gh)\varphi_{ss} = m_s\,a_y\,h, \quad (87)$$

where φ_{ss} is the steady state roll angle. By rewriting Equation (87):

$$\frac{\varphi_{ss}}{a_y} = \frac{m_s h}{k_t - m_s gh} \quad , \tag{88}$$

where φ_{ss}/a_y is the roll gain with a normal range of 4–8 deg/g. By using Equation (88), the vehicle torsional stiffness is found as:

$$k_t = \frac{m_s h}{\varphi_{ss}/a_y} + m_s gh. \tag{89}$$

To formulate the vehicle torsional stiffness, consider a vehicle during cornering: the vehicle body rolls about the roll axis and the outer spring compresses as the inner spring expands. The spring force direction and the spring deformation Δ are shown in Figure 4.39.

For a small roll angle, the spring deformation is determined as:

$$\Delta \approx \frac{\varphi T_s}{2} \quad . \tag{90}$$

The generated roll momentum by the springs of an axle is:

$$M_t = F_s\, T_s\, . \tag{91}$$

where the spring force F_s is:

$$F_s = k_s\, \Delta. \tag{92}$$

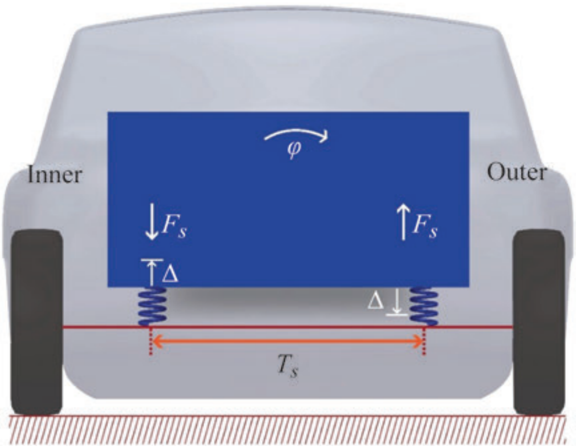


Figure 4.39: Spring force and deformation caused by body roll.

By substituting Equation (90) in Equation (92):

$$F_s = k_s \frac{\varphi T_s}{2}, \tag{93}$$

and by substituting Equation (93) in Equation (91):

$$M_t = k_{ta}\, \varphi, \tag{94}$$

where

$$k_{ta} = \frac{1}{2} k_s\, T_s^2, \tag{95}$$

the total torsional stiffness of the vehicle is calculated as:

$$(k_t)_{vehicle} = \left(\frac{1}{2} k_s\, T_s^2\right)_f + \left(\frac{1}{2} k_s\, T_s^2\right)_r + (k_t)_{ANR_f} + (k_t)_{ANR_r}\,. \tag{96}$$

The total torsional stiffness of the vehicle is the summation of the spring torsional stiffness and the anti-roll bar torsional stiffness for the front and the rear axles. Using an anti-roll bar in the rear axle is not common; therefore, by assuming the rear anti-roll bar stiffness is zero, the front anti-roll-bar stiffness is the only unknown variable in Equation (96):

$$(k_t)_{ANR_f} = (k_t)_{vehicle} - \left(\frac{1}{2} k_s\, T_s^2\right)_f - \left(\frac{1}{2} k_s\, T_s^2\right)_r\,. \tag{97}$$

Example 7: Determine the front anti-roll bar torsional stiffness of a vehicle with the following data:

- vehicle mass m = 1,340 kg;
- vehicle sprung mass m_s = 1287 kg;
- height of vehicle center of gravity H = 0.506 m;
- height of vehicle roll axis h_{RC} = 0.335 m;
- distance between the front axle and the center of gravity a = 1.135 m;
- distance between the rear axle and the center of gravity b = 1.415 m; and
- front and rear spring stiffness k_{sf} = 20 kN/m and k_{sr} = 22 kN/m,

Distance between the springs on the front and rear axles T_{sf} = 1.115 m and T_{sr} = 0.925 m.

Solution: Distance between the vehicle central of gravity and the vehicle roll axis is:

$$h = H - h_{RC} = 0.171 \text{ m}.$$

The roll gain is supposed to be 6 deg/g, so:

$$\frac{\varphi}{a_y} = 6\,\frac{deg}{g} \times \frac{\pi\ rad}{180 \text{ deg}} \times \frac{1\ g}{9.81\ m/s^2} = 0.01\,\frac{rad.s^2}{\text{m}}.$$

By using Equation (89), the vehicle torsional stiffness is calculated as:

$$(k_t)_{front} = \frac{m_s h}{\varphi_{ss}/a_y} + m_s\, gh = \frac{1287 \times 0.171}{0.01} + 1287 \times 9.81 \times 0.171 = 24167 \text{ N.m.}$$

The front and rear torsional stiffness are:

$$(k_t)_{front} = \frac{1}{2}\, k_{sf}\, T_{sf}^2 = \frac{1}{2} \times 20000 \times 1.115^2 = 12432 \text{ N.m,}$$

$$(k_t)_{rear} = \frac{1}{2}\, k_{sr}\, T_{sr}^2 = \frac{1}{2} \times 22000 \times 0.925^2 = 9412 \text{ N.m.}$$

Thus, according to Equation (97), the front anti-roll bar stiffness is:

$$(k_t)_{ANR_f} = (k_t)_{vehicle} - \left(\frac{1}{2}\, k_s\, T_s^2\right)_f - \left(\frac{1}{2}\, k_s\, T_s^2\right)_r = 2323 \text{ N.m.}$$

Author Biographies

Dr. Avesta Goodarzi has more than 20 years of experience in the field of automotive engineering in both the academic and industrial sectors. He is a specialist in the field of Automotive Design and Testing, Vehicle Mechatronics and Control Systems, Vehicle Dynamics, and Electric and Hybrid Electric Vehicles. He was an automotive designer for eight years in world-class automotive research and testing centers. Later, as a university professor, he performed many advanced researches and has been the author of a book and more than 80 published technical papers in prestigious journals and international conference proceedings, all about automotive engineering. Currently, he is the founder and CEO of CARmonizer.com.

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